

# Pricing of Index Options Using Black's Model

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## Abstract

Stock index futures sometimes suffer from 'a negative cost-of-carry' bias, as future prices of stock index frequently trade less than their theoretical value that include carrying costs. Since commencement of Nifty future trading in India, Nifty future always traded below the theoretical prices. This distortion of future prices also spills over to option pricing and increase difference between actual price of Nifty options and the prices calculated using the famous Black-Scholes formula. Fisher Black tried to address the negative cost of carry effect by using forward prices in the option pricing model instead of spot prices. Black's model is found useful for valuing options on physical commodities where discounted value of future price was found to be a better substitute of spot prices as an input to value options. In this study the theoretical prices of Nifty options using both Black Formula and Black-Scholes Formula were compared with actual prices in the market. It was observed that for valuing Nifty Options, Black Formula had given better result compared to Black-Scholes.

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**Index terms**— Options Pricing, Cost of carry, Black-Scholes model, Black's model.

## 1 Introduction

It is generally assumed that the relation in prices between the underlying assets and the futures is maintained by arbitrageurs. If this relation is maintained effectively, then investors find these markets as perfect substitutes, and their choice of trades in these markets are governed by conveniences and costs. However, many studies have reported substantial deviation between futures prices from their theoretical values.

Prices of index futures quoting below the theoretical prices of the stock index futures are a common phenomenon. Since the inception of Nifty future trading in India, Nifty futures even traded below the Nifty spot value. Future prices quoting below spot prices are sometimes observed in commodity prices. Even though Future prices are less than spot prices, the owners of physical commodity may not be able to sell the commodity at the current higher prices and exploit the price differential due to certain constraints. In cases of certain agricultural crops, spot prices increase before harvest due to shortages but just after the harvest prices are reduced when fresh supplies are available. These deformities can cause difference between actual price of options and the prices calculated using options pricing formulas.

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## 2 II.

## 3 Literature Survey

A number of studies are available where differences between theoretical and actual option prices and arbitrage trading opportunities are explored. The arbitrage process is based on the existence of a frictionless market in which traders can make transactions whenever prices of options and futures contracts deviate from their respective fair values. The arbitrage process are sometimes affected by various frictions such as: regulatory restrictions and barriers, transaction costs, risks from the arbitrage process, regulatory restrictions, operating restrictions such as margin size and short selling constraints, etc.

These factors are responsible for an arbitragefree band around the futures and the options prices and minor deviations of pricing within the band can not be used for profitable trades. A number of papers have observed significant variation in the option and futures prices from their fair value relative to the underlying asset. In some cases the deviations may not be sufficient to generate risk less arbitrage and significant profits after costs are taken into account. ??ornell and French (1983) studied stock index futures pricing and arbitrage opportunity with daily data by using the cost of carry model and found that mispricing did existed. Modest and Sundaresan (1983) found that when arbitrageurs lose the interest earnings on the proceeds of the short sale of stocks then pricing band would be asymmetric around the fair price. However, presence of various restrictions will discourage taking short positions in futures; they generally sell stock they own or control when the futures are underpriced.

Fisher Black (1976), one of the co-author of the famous Black and Scholes model (1973) attempted to address this problem of negative cost of carry in the option pricing model by using forward prices instead of spot prices. He found that actual forward prices also capture other irregularity faced by market forces in addition to the inventory carrying cost. Black's model is found useful for valuing options on physical commodities where negative cost of carry is common.

In this study we estimated option prices using both Black's formula and Black-Scholes formula and compared these theoretical values with actual prices in the market and observed that Black formula gives better result in comparison to Black-Scholes formula for Nifty options.

MacKinlay and Ramaswamy (1988) carried out studies using the cost of carry pricing involving the S&P 500 contract from June 1983 to June 1987, and examined the mispricing of futures contracts from their fair values. They found that a positive or negative persistence in mispricing existed. They used regression analysis and found that the mispricings are a function of the average absolute daily mispricing and the time to expiration of the futures. Further analysis of the arbitrage violations led them to conclude that once an arbitrage band was crossed it was less likely for the mispriced value to cross the opposite arbitrage band. They did not provide information on the size of the average arbitrage profits.

Stoll and Whaley (1988) studied the impact of similar strategy on the index futures using a forwardcontract pricing relationship. Lee and Nayar (1993) have studied the S&P 500 options and futures contracts traded on the Chicago Board Options Exchange (CBOE) and Fung and Chan (1994) analyzed trading in the Chicago Mercantile Exchange (CME), respectively to detect pricing efficiency and detected presence of mispricing.

Yadav and Pope (1994), studied the UK markets examining futures against the index and reported that the absolute magnitude of mispricing often exceeds the estimated trading costs and cannot be explained by dividend uncertainty, marking to market cash flows, or possible delays in trade execution. Sternberg (1994) observed that the options contracts available against futures reduce the mispricing since the options can be priced directly against the underlying futures contract.

Traders in options frequently use futures contracts to hedge their positions. Fleming, Ostdiek, and Whaley (1996) found that dealers price the S&P 500 index options based on the prevailing S&P 500 futures price. They found that it is cheaper and more convenient to hedge the options with the futures than with the stock basket. Fleming, Ostdiek and Whaley (1996) analysed trading of S&P 500 futures and found that S&P 500 futures prices appear to lead the S&P 500 stock index, even after controlling for the effects of infrequent trading on the index. The structure of trading costs revealed that futures trades had permanent price effects as traders prefer to exploit their information advantages in the futures market rather than in the cash market.

Traders and market makers often value index options based on the prices of index futures than the spot price of the index. Gwilm and Buckle (1999) observed that the use of the Black's formula of option pricing to price European index options gave better results when delta hedging was possible in futures market. They tried to relate mispricing of the index futures with the mispricing in the index options using Black's formula. ??erma (2003) observed that Nifty futures trade at a discount to the underlying and credited this occurrence on the short sale restrictions in the cash market and estimated the implied (risk neutral) probability distribution of the underlying index using the Breeden-Litzenberger formula.

Berkman, Brailsford and Frino (2005) used the FTSE 100 stock index futures contract and found that there was a small permanent price impact associated with trades in index futures. Their results revealed that the initial price reaction is reversed soon. They did not find evidence of asymmetry in the price reaction following large trades in stock index futures, and suggested that the asymmetry documented in previous studies was specific to equity markets.

Majority of the studies referred above found evidence of mispricing in both futures and options market. It was also found that mispricing in one instrument influence pricing of other instrument. Black's formula paves a way to connect mispricing of futures in option price estimation III.

## 4 Pricing Of Futures With Cost Of Carry

The cost of carry is calculated taking the difference between the actual future prices and the spot value of the underlying asset. The cost of carry concept is based on the cost involved in holding the asset for the validity period of the future contract that include cost of funds blocked, storage, insurance and other handling costs incurred in acquiring and storing the commodity. For financial assets, the cost of carry is measured as equal to the interest rate in a risk free asset. It should be emphasized that it is difficult to formulate a model for the

deviations of futures prices from fair values. Consider the following portfolio, bought today and held until the expiry of forward contract at date  $t$ :

? Buy the stocks at the price  $S$  (the current price) and reinvest the dividends received, if any, until date  $t$ .  
 ? Borrow an amount  $S$  today to finance the purchase of stocks. ? Sell a future contract at the current forward price  $F$ .

To create an arbitrage free situation that is to avoid losses or gains in the above action, following cost of carry based future pricing model can be established.

$F$  is the forward price,  $S$  is the spot price,  $r$  is the risk-free rate of interest,  $H$  is the cost of holding the asset,  $d$  is the dividend or income from the asset during the holding period and  $t$  is the duration of the forward contract (expressed as a fraction of 1 year). Generally, forward prices follow cost of carry arbitrage. When a commodity can be stored, the forward price of that commodity is the cost of purchasing the commodity and holding costs of keeping the position until the forward delivery date. However some products like natural gas, perishable commodities, etc do not follow cost of carry model as such products can not be stored. In case of financial assets, cost of carry is the cost of financing the position that includes cost of funds blocked. In general, the cost of carry is 'positive' as a result of positive interest and storage costs, but in some situations it can also be negative.

The main factors that influence differences between commodity and equity index futures are: ? There are no costs of storage involved in holding equity (depository costs are negligible). ? Equity offers a dividend income, which is a negative cost when stocks are held and positive when stocks are short sold. Therefore, Cost of carry for a financial product is equal to financing cost -dividend income. When the future price at market  $F$  is greater than  $S - rt$  then a strategy of buying the index and selling the future contracts will earn risk less profits in excess of the riskfree rate, similarly when  $F$  is less, then a strategy of selling index and buying futures contracts will achieve financing below the risk-free rate.

Trading in the stock index derivatives has become popular as they are useful to hedge equity portfolios against market fluctuations. The transaction costs in the derivatives markets are lower compared to the trading costs in the cash market. Further, the securities available for trade in the cash market sometimes exhibit illiquidity and higher bid-ask spread. Additionally, short sales of index futures are easy but short sales of securities are restricted in many markets.

For futures involving Nifty index, it was found that future prices were lower than the cost of carry model. When a under priced stock index future is available, the stock index future can be purchased but selling the constituents of the stock index is difficult as there are restrictions on short sales in the spot market. Thus, at the time when market opinion is bearish, anyone can easily take a short position in futures market but can not do so in spot market. When sellers outnumber buyers, the index future price declines and, sometimes go below the spot prices. Since this under pricing can not be exploited by the arbitrageurs these mis-pricing persists for a long time. On the contrary, when Nifty futures are overpriced the arbitrageurs can sell nifty futures and buy underlying stocks from the spot market and take advantage of the mis-pricing. These operations would diminish the arbitrage opportunity quickly and overpricing of futures would soon be corrected. The logic makes it clear that under pricing of Nifty futures can persist but overpricing cannot.

As both futures and options are traded in the same exchange wrong pricing of one instrument are bound to influence pricing of the other and accordingly under pricing of Nifty options are certainly to have an effect on pricing of Nifty options. Black observed that under priced futures could result in under priced call options and overpriced put options. The effect can be termed as the low-call-high-put bias in option prices due to under pricing of futures.

#### IV.

## 5 Black-Scholes Model And Black's Modification

The Black and Scholes (1973) model of options pricing, was a significant development in theoretically estimating the option pricing problem. The Black-Scholes model is attractive since it gives a closed-form solution for pricing of European options. With the sole exception of volatility measure, all other variables used in the model are observed from the market and therefore the model had contributed to the expansion of the options markets as a effective pricing technology were made available. Though the original model was developed for non-dividend paying securities in European type options, the model can be modified for the pricing other types of options. The Black-Scholes formulas for the prices of European Calls (C) and Puts (P) for no dividend paying stocks are given below.)

In this formula  $S$  = current price of the security  $X$  = Exercise price of option  $r$  = Risk free rate of interest  $t$  = Time to expiry of the option content  $\sigma$  = Volatility of the underlying asset  $N(x)$  is the cumulative probability function for a standardised normal variable. It represent the probability that a variable with a standard normal distribution  $\Phi(0,1)$  will be less than  $x$ .

A number of assumptions used in the Black-Scholes method appears to be unrealistic. First, it assumes a geometric Brownian motion of stock prices where the series of first differences of the log prices must be uncorrelated. But actual returns from stocks exhibit small but statistically significant correlations in the differences. The next question is whether the innovations returns are normally distributed. It is also a common observation that the returns are leptokurtic, i.e., they have much more of a tendency to exhibit outliers than would

be possible from a normally distributed series. Finally, the assumption of constant variation is also questionable. The levels of volatility (i.e., fluctuation) follow immediately after a large change in the level of the original prices and high volatility usually persists for some time. When the underlying assumptions are violated, the use of Black-Scholes formula to compute options prices may not always be accurate.

In spite of the violations mentioned above, Black-Scholes model is still very widely used, but sometimes adjustments are made to account for some of the identified inadequacy.

## 6 a) Black's Formula

The original Black-Scholes model has undergone several theoretical developments. One such development for the valuation of futures options is introduced by Black (1976). Black proposed a formula for options under the assumption that investors generate risk less hedges between options and the futures or forward contracts. The problem of negative cost of carry was addressed by using 'forward prices' in the option pricing model instead of 'spot prices'. Black observed that actual forward prices not only incorporate cost of carry but also takes into account other irregularities in the market. In his proposed model, he substituted spot price (S) by the discounted value of future price (F.e-rt) in the original Black-Scholes model. Black's model found application in valuing options on physical commodities where future price is a better alternative input for valuing options.

The Call options prices as per Black's formula can be observed solving following equation:

$$C = \left( \frac{F}{e^{rt}} \right) N(d_1) - \left( \frac{K}{e^{rt}} \right) N(d_2) \quad (6.1)$$

Where  $d_1 = \frac{\ln(F/K) + (r + \frac{1}{2}\sigma^2)t}{\sigma\sqrt{t}}$  and  $d_2 = d_1 - \sigma\sqrt{t}$

In the formula F is the future price of the asset and other input parameters are similar to the inputs used in the Black-Scholes model.

According to Black, future prices provide valuable information for the market participants who produce, store and sell commodities. The future prices observed during the various transaction months, help the producers and traders to decide on the best times to plant, harvest, buy and sell the physical commodity. The future price of a commodity therefore reflects the anticipated distribution of Spot prices at the time of maturity of the future contract. Black observed that changes in spot price and change in future price are usually correlated. Both spot prices and future prices are governed by the general shifts in cost of producing a commodity and the general shifts of demand of the commodity. Shifts in demands and supply due to fresh harvesting can create difference between spot and future price.

In the Black-Scholes formula, the term  $X e^{-rt}$  represents the present value of exercise price discounted at risk free rate r for the time to maturity. The expression is based on the premise that the exercise price of the option at a future date includes an interest rate component over the intrinsic value of exercise price. In the same logic, the future prices are supposed to be higher than the spot price due to the positive interest rate component. Thus the important difference between Black's and Black-Scholes in that Black uses forward prices and Black-Scholes uses spot prices.

The Nifty future prices are usually lower than their theoretical prices. The main reasons of futures trading less than their fair value are due to the short selling restrictions of underlying stocks. When future price is higher than the spot value plus cost of carry (i.e.  $F > S.e^{rt}$ ) one can sell future and buy underlying stocks, on the other hand when future price is less than the spot value plus cost of carry, (i.e.  $F < S.e^{rt}$ ) stocks can not be short sold on account of short sell restrictions. As a result of this restriction in India, future prices time and again trade less than their fair values. It was observed that all the futures of Nifty index were trading at prices less than their intrinsic values and in many instances the futures were traded even below their spot prices.

V.

## 8 Data And Analysis

Call options of Nifty index traded in National Stock Exchange of India from the period 1st July 2008 to 30th June 2011 were collected from website of the exchange: [www.nseindia.com](http://www.nseindia.com). Similarly, closing prices of the Index for the said period were also gathered. Since only closing prices of the day were available with the exchange, inter-day prices could not be compared. The comparison of closing prices can give error for thinly traded options due to mismatch of timing. For example, when an option was traded last at 1.00 pm, the spot price of the same asset was different than the closing price taken at the close of the day. Thus the often change with time. The periods of high volatility closing price of the asset can not be used to evaluate a trade that took place much earlier. With the availability of data from the exchange, comparing call option prices traded at different times with the corresponding spot values at that time were not possible. This effect of timing mismatch was reduced by short listing only highly traded options that were traded till last few minutes and hence options where the volume of trading was higher than 100 lots in the day were selected for the study.

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## 9 a) Under pricing of Nifty futures

In this study Nifty futures traded during the period July 2008 to June 2011 were analyzed. Out of 743 days observation, it was found that in case of 603 days (80.89% of the days) corresponding futures prices were lower their fair value (i.e. spot price plus cost of carry) (table-1). This bias is bound to influence options pricing in the options market. The future prices quoting lower than their fair value were common during the sample period.

In many days the future prices even quoted below their corresponding spot values. Out of 743 days, as many as 271 days (36.47% of the cases) futures prices were below their corresponding spot prices.

## 10 b) Other parameters used in option Pricing

Other input parameters for estimating call option prices with the formulas are obtained as follows. Among input parameters required in the models, the standard deviation (?) of the returns for the duration of the option are not observable from the market and therefore an estimate is required. There is no agreement on the suitable method for estimating standard deviation of the returns series. Further, it is a common observation that '?' of the price series changes with time. Sometime the returns remain volatile and this volatility persists for some time and again the volatility may remain low in other periods.

The past volatility of a security can be estimated as the standard deviation of a stock's returns over a predetermined number of days. Choosing the appropriate number of days is complicated. Longer period of observation has an averaging effect and as volatility varies with time and very old data may not be relevant for the current situation and can not be used for predicting the future. In absence of an agreed method to estimate volatility to be used in options pricing models, a simple method of estimating standard deviation using past three months return was used in the study.

According to Hull (2004), use closing daily prices of few recent months a compromise solution. The daily volatility can be annualized as follows.

. The number of trading per annum are usually taken 250 or 252 that exclude weekly offs and holidays.

The value of risk free rate of interest was taken at 8% as small investors can earn this rate from post office savings schemes and similar government backed savings.

## 11 c) Comparison of errors using option pricing models

After gathering the required data call option prices using both Black-Scholes formula and Black's formula were estimated and compared. The easiest way to measure efficacy of a valuation formula was to compare the calculated values with the option prices quoted in the market. Any difference between actual price and calculated value was taken as the error of the formula. The formula that had produced lowest error was considered better. The error for each observation was added to obtain total error. To find effectiveness of the models, a paired sample t-test was carried out involving error measures of the two methods. The Paired Samples t-test compares the means of two variables. It computes the difference between the two variables for each case, and tests to see if the average difference is significantly different from zero. The following hypothesis was tested:

? Null: There is no significant difference between the means of the two variables.

? Alternate: There is a significant difference between the means of the two variables In the test the mean of error measures between Black's method and Black-Scholes method were compared to verify if the use of Black's method improved accuracy of estimating call options. The test was carried out in SPSS and the descriptive statistics for both the error measures are presented in table ???. In the table the difference between the pair of variables is given. The mean error of the Black method was 1.76 and the mean of the B-S model was -10.49. The results of the significant tests are produced in table ??, where the difference between two means were 12.25 and the result was significant with the value of  $p = .000$ . It is therefore concluded that the differences of the mean was significant and it is more likely to obtain better result using Black's method compared to Black-Scholes method.

## 12 VI.

## 13 Conclusion

The purpose of the study was to evaluate the accuracy of option pricing models to value Nifty Index Futures traded on National stock exchange of India. Though the use of Black-Scholes model is popular, the model does not exactly fit into the real life situations. Index futures suffer from the negative cost of carry effect and Nifty futures quoting below the theoretical prices of Nifty are very common. Since the beginning of Nifty trading in India, Nifty futures remained under priced and sometimes trade below the Nifty spot value. This type of mismatch in pricing of futures is usually seen in commodity futures. When future prices are lower, the owners of the commodity may not be able sell the product due to various reasons and benefit from the mismatch in pricing.

The paper tried to address issues related to under-pricing of Nifty options on account of negative cost of carry in futures market. In this study, 29,724 option quotes from 1st July 2008 to 30th June 2011, are analyzed using both B-S formula and Black formula and found that the Black's formula produce better alternative than use of Black and Scholes formula.. From the analysis of errors, it is verified that Black model produces less error than

## 13 CONCLUSION

that of Black-Scholes model and for that reason use of Black model is more fitting than that of B-S model for valuing Nifty options. <sup>1 2</sup>

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Figure 1:

1

		Period		No. of		% cases where	
Sl	From	To	Total	Observa- tion F<S ert	F>S ert	F<S ert	
1	1-Jul-08	30-Sep-08	64	39	25	60.94%	
2	1-Oct-08	31-Dec-08	60	42	18	70.00%	
3	1-Jan-09	31-Mar-09	59	59	0	100.00%	
4	1-Apr-09	30-Jun-09	59	47	12	79.66%	
5	1-Jul-09	30-Sep-09	64	57	7	89.06%	
6	1-Oct-09	31-Dec-09	61	53	8	86.89%	
7	1-Jan-10	31-Mar-10	60	57	3	95.00%	
8	1-Apr-10	30-Jun-10	63	59	4	93.65%	

Figure 2: Table 1 :

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		Period		No. of Ob- servation		% cases where
Sl	From	To	Total	Future<Spot	Future>Spot	Future<Spot
1	1-Jul-08	30-Sep-08	64	18	46	28.13%
2	1-Oct-08	31-Dec-08	60	22	38	36.67%
3	1-Jan-09	31-Mar-09	59	51	8	86.44%
4	1-Apr-09	30-Jun-09	59	14	45	23.73%
5	1-Jul-09	30-Sep-09	64	29	35	45.31%
6	1-Oct-09	31-Dec-09	61	26	35	42.62%
7	1-Jan-10	31-Mar-10	60	24	36	40.00%
8	1-Apr-10	30-Jun-10	63	34	29	53.97%
9	1-Jul-10	30-Sep-10	65	19	46	29.23%
10	1-Oct-10	31-Dec-10	64	1	63	1.56%
11	1-Jan-11	31-Mar-11	62	15	47	24.19%
12	1-Apr-11	30-Jun-11	62	18	44	29.03%
	Total		743	271	472	36.47%

Figure 3: Table 2 :





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