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Said Sami Al Hallaq [α], Mohamad Mahmoud Ajlouni [σ] & Mahmoud Faheem Al Saadi [ρ]

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I. Introduction

Selecting investment portfolio is one of the most important research areas in modern finance; it seeks to better allocate funds between baskets of securities. Portfolio selection was first introduced by "Harry Markowitz" in 1952, in his paper "portfolio selection." He explained the concept of diversification, and suggested that investors focus on portfolio selection depending on their overall risk-reward characteristics.. The Markowitz model consists of only two factors, which are the expected return and variance, and presumes investors are risk averse. The idea of the model is that an investor cannot achieve a high return without increasing portfolio risk.

Following Markowitz, many attempts have been conducted in the portfolio management to find new mathematical approaches to select the optimum portfolio. The computational capacity of the 21st century and the wide availability of computers have allowed the development of a new generation of intelligent computer techniques such as hybrid genetic algorithm (HGA). HGA is a problem-solving technique motivated by biological evolution, based on an artificially simulated natural selection process or the survival of the fittest, known as Darwinian evolution. HGA can only identify the portfolios being in the efficient frontier and has been used in many recent works to find the optimum portfolios, due to multiple objective functions in portfolio selection. Therefore, the Analytic Hierarchy Process (AHP) approach is tied to the HGA to select the optimal portfolio based on all criteria between the portfolios obtained by HGA. The main objective of this study is to investigate the ability of HGA and AHP in selecting the optimum portfolio using data of the Jordanian companies listed on the Amman Stock Exchange (ASE).

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a) *Study Questions*

- Can HGA and AHP be used in selection of optimum portfolio?
- Can Efficient Frontier (EF) be identified using HGA?
- Can AHP select the optimum portfolio among the portfolios obtained by HGA?

b) *Objectives of the Study*

The main objective of this study is to investigate the ability of hybrid genetic algorithm and analytic hierarchy process in selecting the optimum portfolio. In particular, the study aims to identify the efficient frontier using HGA and selecting the optimum portfolio according to AHP from HGA's EF.

c) *Importance of the Study*

A review of the literature on the optimum portfolio selection using a hybrid genetic algorithm and analytic hierarchy process indicates that most of the studies on this topic have been conducted in developed countries. This is probably attributed to the fact that such studies require a relevant level of data disclosure and analyzing techniques. Thus, this study will contribute to the literature by producing answers regarding the selection of optimum portfolio using a hybrid genetic algorithm and analytic hierarchy process. HGA and AHP are new techniques, which are added to portfolio selection to reach EF and optimum portfolio. Therefore, the importance of this methodology is derived from the importance of optimum portfolio, which all investors wish to reach.

d) Structure of the Study

The paper is organized as follows: In addition to the introduction, section two presents the theoretical background of the study. Section three covers the literature review, and what distinguishes the current study. Section four describes the methodology used. Section five presents the results. The final section presents recommendations and policy implications.

II. Literature Review

The beginning of the modern investment theory is traced back to 1952, when Harry Markowitz (1952) published an article titled "Portfolio Selection." He showed how to create a frontier of investment portfolios, so that everyone had the highest expected rate of return given its level of risk or the minimum level of risk given its rate of return. The calculation technique was very complex, especially given the technology of the time. In the past, the optimization of Markowitz's portfolio was used, mostly in the asset allocation decision. The investor decides on the amounts to invest in certain basic classes such as stock, bonds and real estate assets. The computing power needed to optimize more than a few asset classes is only a small fraction of what is needed to optimize more than thousands of stocks. There is a need to utilize the available quantitative data to solve the optimization problem. Prior to the spread of portfolio theory in the real world, three scholars simultaneously and independently asked the following questions: Assume that everyone successfully makes his investments using portfolio theory and invested in portfolios on the frontier, how would this affect the price of securities? In response to this question, Sharpe (1964), Lintner (1965) and Mossion (1966) developed the Capital Asset Pricing Model (CAPM), which is widely used in the real world to measure portfolio performance; securities value, make capital budgeting decisions, and even regulate utilities. However, the model was challenged by Richard Roll (1977, 1978), who argued that the model should be discarded because it was impossible to verify empirically its single economic forecast. This controversial issue remains the subject of a lively debate today. At the same time, Steve Ross (1976) developed an alternative model to CAPM. This model was called the Arbitrage Pricing Theory (APT) where expected return should be linked to risk so that no single investor could create unlimited return through arbitrage. The question of how to price option contracts has puzzled researchers in finance until a paper of Fisher Black and Myron Scholes was published in 1973. They argued that you could make a riskless hedged position with an option by taking a position in both the option and the stock it is written on. Although researchers in the finance science were trying to determine the nature of the price structure in the securities markets, the issue of how efficient the market is in pricing to its structure was called into question. Fama (1970) stated that a market is efficient if security prices immediately and fully reflect all available relevant information. Fama, (1991) divided the overall EMH and the empirical test of the hypothesis into three sub-hypotheses depending on the information set including: Weak-Form of Efficiency, Semi Strong-Form of Efficiency, and Strong-Form of Efficiency (Haugen, 2001).

The Markowitz theory is now known as the Modern Portfolio Theory (MPT).. Although MPT is widely used in practice in the financial sector in recent years, the basic assumptions of the same have been largely questioned. MPT, as an improvement over traditional investment models, is an important mathematical modeling for the advancement of finance.

a) Optimal Portfolio Selection

Portfolio theory assumes that investors are essentially risk averse, meaning that given the choice between two assets with equal rates of return, they would select the asset with the lowest risk (Maginn *et al*., 1990). This combination of risk preference and risk aversion can be explained by an attitude to risk that depends on the amount of money involved. Although diversity of attitudes is recognized, the basic assumption is that most investors who use large sums of money to develop an investment portfolio are averse to risk. As a result, there is a positive relationship between expected return and expected risk in the optimal selection process (Peavy, 1990). The optimal portfolio is a combination of investments, each of which has desirable individual risk-return characteristics that are also adjusted according to their correlations (Desai *et al.,* 2003). The optimal portfolio is the efficient frontier portfolio that has the greatest value for a given investor. It is at the point of tangency between the curve of the efficient frontier and the curve with the highest potential utility.

To expand Markowitz's model portfolio and the assumptions of EMH the risk-free rate of return should be considered. Correlation and covariance of any asset with a risk-free asset is zero. So any combination of an asset or portfolio with the risk-free asset generates a linear return and risk function. As a result, the combination of the risk-free asset with a risky asset in the Markowitz efficient frontier gives rise to linear portfolio opportunities while the dominant line is that which tangents the efficient frontier. This dominant line is known as the Capital Market Line (CML) (Haugen, 2001). Because all investors want to invest in the portfolio that is risky at the point of tangency, this portfolio - known market portfolio must contain all risky assets pro rata to their relative market values. In addition, the investment decision and financing decision can be separated because, although everyone wants to invest in the market portfolio, investors make different financing decisions on loans or borrow depending on their preferences of individual risk (Davis and Norman, 1990). Given the CML and the predominance of the market portfolio, the relevant risk measure for an active individual risk is its covariance with the market portfolio. That is, it is systematic risk, when the covariance for the market portfolio is standardized, a known beta measure of the systematic risk and market line, which relates to the expected or required rate of return of an asset with its beta version (Lee and Su, 2014). Individual securities and portfolios are represented in the Security Market Line (SML) to determine the expected return because of a systematic risk (beta). Alternatively, assuming that markets are not always fully effective, one can identify undervalued and overvalued securities by comparing the estimated rate of return of an investment to its expected rate of return. The systematic variable risk (beta) for an active individual risk is calculated using a regression model that generates an equation referred to as the assets characteristic line (Hong and Sarker, 2007).

III. Data and Methodology

The study population consists of all companies listed in Amman Stock Exchange (ASE) during the period of January 1, 2015 to December 31, 2015. At the end of 2015, the total number of companies listed was 228 companies. The current study sample has been selected according to the criteria of continuous data availability. Therefore, the sample is limited to companies that met the following criteria:

1) Companies should have complete data availability during the study period.
2) Companies should have been established before 15 years.
3) Companies should not have been engaged in acquisitions or merger during the study period.

By applying these criteria, the final sample of this study is restricted to 60 companies. As for the data the study depends on secondary data that have been published in annual reports issued by Jordanian companies listed on Amman Stock Exchange (ASE), reports and trading data issued by ASE, statistical databases issued by the Central Bank of Jordan (CBJ), books and references, studies, previous researches related to the subject matter, and network (internet) publications. To achieve the purposes of the study, the following variables will be used: return, risk, beta, liquidity ratio, Sharpe ratio, Treynor's ratio and Jensen's alpha.

a) Mathematical Model

The current study aims to test the ability of HGA and AHP in selecting the optimum portfolio of shares listed in Amman Stock Exchange. The model used in this study is based on the Yin-Wing and Yuping (2000) and Solimanpur *et al.* (2015) models.

Notations: The following notations are used to formulate portfolio selection problem:

- I: Index for stocks.
- N: total number of stocks.
- ri: Return of stock i.
- Rp: Return of portfolio.
- σp: Risk of portfolio
- xj: Percentage of stock j in portfolio.

Objective Functions: The attempted mathematical model includes two objective functions. Namely, return and risk of portfolio. These objective functions are formulated based on the following:

- *Return of Portfolio:* The portfolio return is defined as the weighted average returns of the portfolio shares and is expressed as follows (Haugen, 2001):

$$R_p = \sum_{i=1}^{N} r_i x_i \quad \ldots\ldots \qquad \ldots\ldots\ldots\ldots\ldots\ (1)$$

Risk of portfolio: Risk of a stock is defined as the tolerance of stock's return from the mean. The risk of a portfolio is expressed as follows (Haugen, 2001):

$$\sigma_P = \sqrt{\sum_{i=1}^{N} \sum_{j=n}^{N} r_i x_j \, cov_{ij}} \qquad \ldots\ldots\ldots\ldots(2)$$

Where:

cov_{ij}: The covariance between returns of stock i and j expressed as $cov_{ij} = r_{ij}\,\sigma_i \sigma_j$.

r_{ij}: is the correlation coefficient between i and j.

Constraints: The attempted model has the following constraint:

$$\sum_{i=1}^{N} x_i = 1; if\ x_i \geq 0, i = 1,2, \ldots, N$$

It should be noted that the non-negativity of the decision variables is used to prevent short selling.

IV. Proposed Hybrid Genetic Algorithm (HGA)

a) Portfolio Display

In the proposed HGA, any portfolio is represented by $N \times num\_$ bits genes in which N is the number of companies and num_bits is the number of binary bits used for representing the share of each company in the portfolio. The share of company i in the portfolio can be calculated by (Solimanpur, 2015):

$$xi = \frac{v_i}{\sum_{i=1}^{N} v_i} \ldots\ldots\ldots\ldots\ldots\ldots \qquad (3)$$

Where:

v_i: The decimal value of the binary code dedicated for company i.

This formula ensures that, after the execution of genetic operations, the constraints of mathematical models will be satisfied.

b) Fitness Function

Fitness of any chromosome in the proposed HGA is calculated in K directions. Fitness of chromosome S in direction k is computed by (Solimanpur, 2015):

$$fit_k(S) = w_{k1}R'_p(S) + w_{k2}\sigma'_p(S) \qquad \text{........... (4)}$$

Where:

$fit_k(S)$: is the fitness value of chromo some S in direction k.

$R'_p(S)$: is the normalized value of return of chromosome S.

$\sigma'_p(S)$: is the normalized value of risk of chromosome S.

w_{k1}: is the weight of return in direction k.

w_{k2}: is the weight of risk in direction k.

The normalized values of return and risk of portfolio S in the population are defined as follows (Solimanpur, 2015):Ω

〖R^'〗_p (S)=R_s/max┬(U∈Ω)⁡〖(R_U)〗... (5)

〖σ'〗_S =min┬(U∈Ω)⁡〖(R_U)〗/σ_S .... (6)

c) Selection

In the proposed HGA, selection probability of chromosome S in direction k is proportional to the quality of this chromosome in the direction k, i.e $fit_s(S)$. In other words, the higher the fitness of a chromosome, the higher should be the selection probability.

d) Portfolio Selection via Analytic Hierarchy Process (AHP)

The method proposed an evaluation procedure comprising of the following steps (Islklar and Büyükzkan, 2007):

Step 1: Identify all the criteria to be taken into account in the choice of the portfolio (evaluation) and to build a hierarchy of decision-making.

Step 2: Calculate weights of criteria using AHP method. These steps are performed in the following subsections.

V. Evaluation Criteria

Selecting the appropriate portfolio of performance measures to provide the necessary information to investors to assess the effectiveness with which they can invest their money is a vital issue. The performance appraisal is primarily related to the determination of how a particular investment portfolio has performed compared to a benchmark comparison. To effectively manage portfolio selection, it is necessary to consider the critical factors that reflect investor behavior and the state of the financial market. The different stakeholders within a decision process can be relatively diverse, with different objectives and conflicts of value systems. In this respect, a key concept is the relationship between risk and return. Each performance index provides a different perspective on the balance between the level of return and the exposure to risk. Therefore, in this study, seven measures, return, risk, beta, liquidity ratio, Sharpe ratio, Treynor's Ratio (TR) and Jensen's alpha (Alpha ratio) have been identified as criteria, which affect the investors decision in portfolio selection.

- *Beta:* Beta measures the price volatility of a share compared to the rest of the market. This measure of risk is defined as (Haugen, 2001):

$$\beta_i = \frac{Cov_{ri,rm}}{Var\,(r_M)} \text{............................ (7)}$$

Where:

$Cov_{ri,rm}$: The covariance between the rate of return of share i and the return of the market or index.

R_M: The rate of return of the market portfolio.

- *Liquidity Ratio:* This ratio indicates the annual liquidity of a stock. Haugen, (2001) derived from it the liquidity of a portfolio:

$$L_p = \sum_{i=1}^{N} L_i X_i \qquad \text{................ (8)}$$

Where:

L_i: The liquidity of share i.

X_i: The percentage of share i in the portfolio.

- *Sharpe Ratio:* Sharpe ratio or RVAR measures the return into the portfolio risk (standard deviation of return) (Sharpe, 1966):

$$RVAR = \frac{\overline{TR}_p - \overline{RF}}{SD_p} \text{............................. (9)}$$

Where:

$\overline{TR}_p$: The mean of portfolio returns in a certain period.

$\overline{RF}$: The mean of non-risk return rate in time set.

SD_p: The standard deviation of portfolio returns in time set.

$\overline{TR}_p - \overline{RF}$: The surplus return of portfolio.

This ratio measures the surplus return of portfolio versus a risk unit. The higher the RVAR of a portfolio, the higher will be its desirability.

- Treynor's ratio (TR)

TR measures the proportion of extra return on beta (Treynor, 1965). The ratio is defined as:

$$TR = \frac{\overline{TR}_p - \overline{RF}}{\beta_p} \text{........ .... (10)}$$

Where:

β_p: The systematic risk of portfolio (beta ratio).

This ratio measures the excess return of portfolio versus portfolio's systematic risk unit.

- Alpha Ratio

Alpha ratio is linked to Treynor's ratio and, hence, it provides a classification that is close to the performance of a portfolio (Jensen, 1968). This ratio is defined as:

$$\overline{\alpha_p} = \overline{R_p} - [\overline{RF} + (RM - \overline{RF})\beta_p] \text{.................} \quad (11)$$

The weakness and strength of a portfolio's performance has two sources: The first is the portfolio manager's ability to select appropriate shares, and the second is their ability to make appropriate decisions over time and to assess threats and market opportunities. Obviously, the manager who considers these aspects will have a better performance in managing the portfolio. The benefit of using this ratio is the possibility to measure $\overline{\alpha_p}$ and β_p at the same time (Haugen, 2001).

VI. The Analytic Hierarchy Process Methodology

AHP includes six basic steps, as summarized below (Islklar and Büyükzkan, 2007; Saaty, 1980):

Step 1: AHP breaks down a complex multi-criteria decision-making problem to several small sub-problems each with a single criterion. Thus, the first step is to break the problem of decision in a hierarchy with a goal at the top, criteria and sub-criteria at levels and sub-levels and alternative decisions at the bottom of the hierarchy.

Step 2: The decision matrix, which is based on Saaty's nine-point scale, is constructed. The decision-maker uses the fundamental scale defined by Saaty to assess the priority score.

Step 3: It involves pair wise comparison of the elements of the constructed hierarchy. The aim is to set their relative priorities with respect to each of the elements at the next higher level.

Step 4: AHP also calculates an inconsistency index to reflect the consistency of decision maker's judgments during the evaluation phase. The inconsistency index in both the decision matrix and in pairwise comparison matrices can be calculated by the following equation (Saaty, 1980):

$$CI = \frac{\lambda_{max} - N}{N-1} \text{..................................} \quad (12)$$

The closer the inconsistency index to zero; the greater will be the consistency of decision-maker's judgments. The consistency of the assessments is ensured if the equality $a_{ji}.a_{jk} = a_{ik}$ holds for all criteria. The relevant index should be lower than 0.10 to accept the AHP results as consistent. If this is not the case, the decision-maker should go back to Steps 2 and 3 and redo the assessments and comparisons.

Step 5: Before any calculation, the comparison matrix has to be normalized. To normalize, each column is divided by the sum of entries of the corresponding column. In that way, a normalized matrix is obtained in which the sum of the elements of each column is 1.

Step 6: The relative values obtained in the third step should satisfy:

$$A\,W = \lambda_{max}\,W, \text{...... ....} \quad (13)$$

Where:

A: Represents the pairwise comparison matrix.
λ_{max} : The highest eigenvalue.
W: Weight of the vector or elements.

If there are elements at the higher levels of the hierarchy, the obtained weight vector is multiplied by the weights of the elements at the higher levels, until the top of the hierarchy is reached. The alternative with the highest weight is finally considered as the best alternative.

VII. Data Analysis

This section shows the descriptive statistics of the study variables, and then examines the ability of the HGA and AHP to select the optimum portfolio. In particular, this section aims to answer the questions and test the hypotheses of this study.

a) *Hybrid Genetic Algorithm Process and implementation*

To implement the previously discussed methodology, we created an application using Visual Studio 2010, and C#.NET as a programming language. As figure (1) shows, the first step in this work is to read the stocks data. In this step, the program reads the stocks data and creates an object for each stock, and then loads the daily data return for each stock. After that, it uses the stocks, data to create 10,000 portfolios, where by each portfolio consists of six stocks. Following that, it uses the daily data return of the assets to calculate the portfolios risk and return. Then it uses the portfolios risk and return to evaluate the fitness of the portfolios and select the top 10 portfolios to be used in the HGA. After that, it uses the top 10 portfolios in the crossover process to generate the second generation of portfolios. Finally, it selects the top 10 portfolios from the second-generation portfolios and draws the efficient frontier.

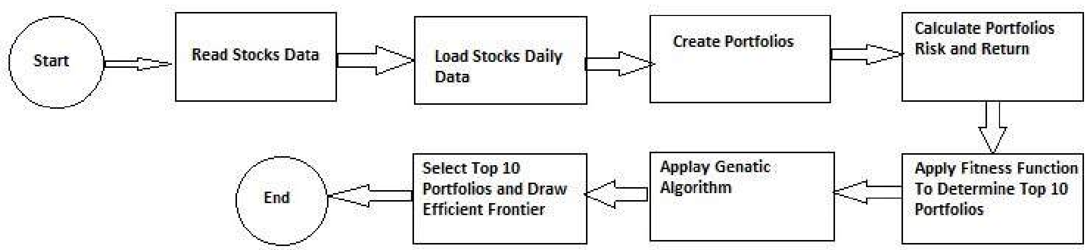

Figure 1: Flowchart of the Implementation Process

b) Implementation Details

The study gathered the stocks data from Amman Stock Exchange website for the year 2015 and used the data to create excel sheets and notepad files which are loaded to the system and an object for each stock is created consisting of five characteristics based on the following:

1) Stock ID: unique integer value varies between 60-120 which is used to identify each stock.
2) Stock Code: Company code as in Amman Stock Exchange.
3) Return: Stock expected return.
4) Risk: Stock risk.
5) Daily Data: The stock daily return.

We have assigned a unique ID for each stock which varies from 60-120. The ID will be used to calculate the weight of each stock in the portfolio, in a manner that the stock with the highest expected return has the highest ID and so on. It would be natural to choose the ID's from 1-60 but we chose to select the ID's from 60-120 for the following reason:

In the 1-60 ID's the share of each stock will dramatically vary between the following values:

Highest weight would be the result of company with ID 60 to be assigned in a portfolio with the stocks of ID's 1,2,3,4 and 5. In this situation the Stock with ID 60 will have a weight of 60/(60+1+2+3+4+5) = 80%.

Lowest weight would be the result of company with ID 1 to be assigned in a portfolio with the stocks ID's 60, 59, 58, 57 and 56. In this situation the Stock with the ID 1 will have a weight of 1/ (60+59 + 58+57+56+ 1)= 0.34%

However, in the situation of ID's 60-120 we had the weights vary between 9.2% and 27.9%. As we can see, the second approach gives a better chance for the assets to affect the portfolio return and risk, and eliminates the possibility of a single asset to dominate the portfolio.

Having created the stocks and loaded the daily data, 10,000 portfolios are created with each portfolio containing six different stocks selected randomly from the previously created 60 stocks. Following the creation of the portfolios the risk and return of each portfolio will be calculated.

Portfolio return will be calculated according to the following formula (Haugen, 2001):

$$Return = w1 * R1 + w2 * R2 + w3 * R3 + w4 * R4 + w5 * R5 + w6\, R6 \quad \text{........ (14)}$$

Where:

W: The weight of each stock in the portfolio.

R: The rate of return of each stock in the portfolio.

The weight for each stock in the portfolio is calculated by dividing the asset ID over the summation of the total assets IDs of the portfolio.

The 6 asset portfolio risk will be calculated according to the following formula (Haugen, 2001):

$$\sigma^2\, por = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + w_3^2\sigma_3^2 + w_4^2\sigma_4^2 + w_5^2\sigma_5^2 + w_6^2\sigma_6^2 + 2w_1w_2\, cov(1,2) + 2w_1w_3\, cov(1,3) + 2w_1w_4\, cov(1,4) + 2w_1w_5\, cov(1,5) + 2w_1w_6\, cov(1,6) + 2w_2w_3\, cov(2,3) + 2w_2w_4\, cov(2,4) + 2w_2w_5\, cov(2,5) + 2w_2w_6\, cov(2,6) + 2w_3w_4\, cov(3,4) + 2w_3w_5\, cov(3,5) + 2w_3w_6\, cov(3,6) + 2w_4w_5\, cov(4,5) + 2w_4w_6\, cov(4,6) + 2w_5w_6\, cov(5,6) \quad \text{........ (16)}$$

And $\sigma\, por = \sqrt{\sigma^2}\, por$

To calculate the covariance we used the stocks daily data to calculate the mean and variance for each stock and used the following formula to calculate the covariance between each two stocks in any portfolio:

$$Cov = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{n} \quad \text{........ (17)}$$

Where:

$\bar{x}$ and $\bar{y}$: are the sample means return averages.

n: is the sample size.

After calculating the risk and return for each portfolio, we apply the following fitness function to calculate the fitness factor for each portfolio (Solimanpur, 2015):

$$fit_k(S) = w_{k1}R'_p(S) + w_{k2}\sigma'_p(S) \quad \text{........ (18)}$$

Where:

$fit_k(S)$: is the fitness value of chromosome S in direction k.

$R'_p(S)$: is the normalized value of return of chromosome S.

$\sigma'_p(S)$: is the normalized value of risk of chromosome S.

w_{k1}: is the weight of return in direction k.

w_{k2}: is the weight of risk in direction k.

To produce our results, we used the following weights for risk and return:

Table 1: Fitness Function Weights.

Weight of risk	Weight of return
50%	50%
40%	60%
80%	20%
20%	80%
60%	40%

After calculating the fitness value for each portfolio, we select the top 10 portfolios based on the highest fitness value.

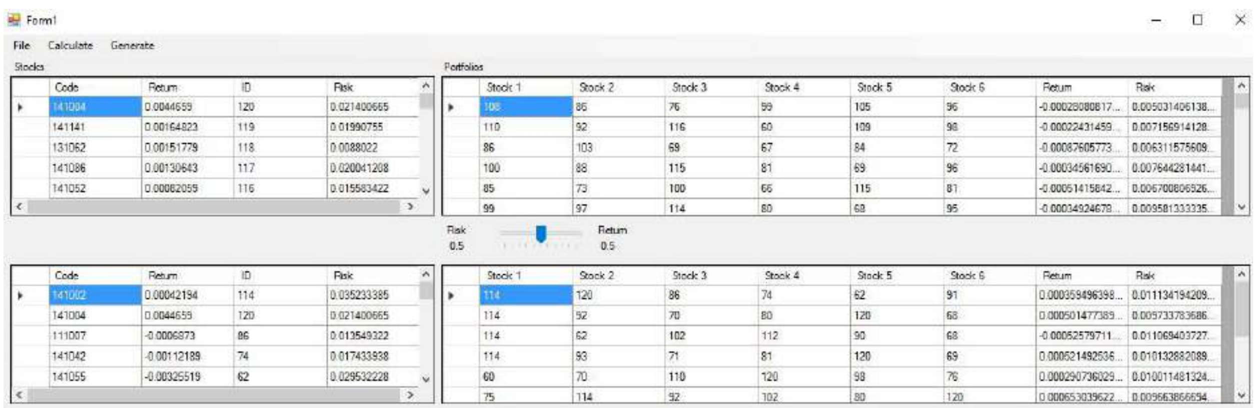

Figure 2: Portfolios and Stocks.

Figure (2) contains the data of the stocks and portfolios. The left top grid shows the stocks and data. The top right grid shows the 10,000 portfolios we have created earlier. In the bottom right grid we can find the assets of which the top 10 portfolios are created. In the bottom right grid, we can find the top 10 portfolios and their associated stocks risk and return.

Now after having determined the top 10 portfolios, it is time to use HGA. In the HGA, each portfolio is treated as a gene, where each gene consists of six chromosomes which represent the assets of the portfolio. A chromosome for a portfolio consists of the following assets {114, 120, 86, 74, 62, 91} would be [1110010 1111000 1010110 1001010 0111110 1011011] where each seven digits represent the chromosome for each asset which are the result of the ID conversion from decimal to binary. The purpose of the HGA is to swap chromosomes between the best genes to generate better chromosomes. To do so, we separated the assets of the top 10 portfolios and used these assets to create the second generation of the portfolios using HGA. After separating the genes from which the chromosomes are created, we used these genes as the basis to create the new portfolios, which is known as the Crossover operation. We have created portfolios six times the number of the successful genes. The created portfolios are referred to as the second-generation portfolios.

After creating the second-generation portfolios, we selected the top 10 portfolios from the second-generation portfolios and created the efficient frontier graph.

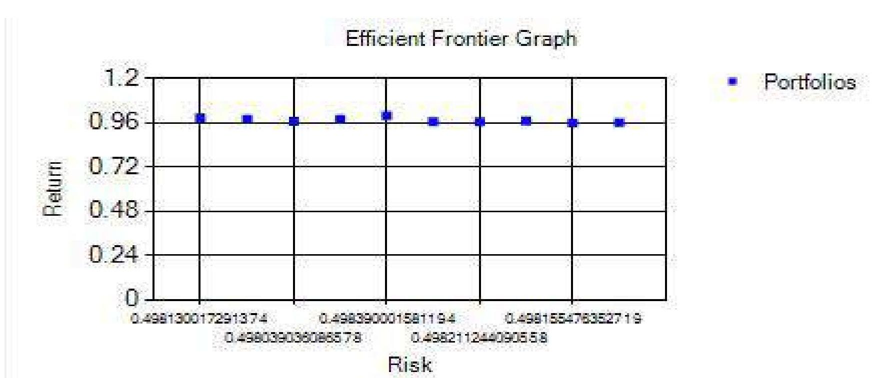

Figure 3: Efficient Frontier

As figure (3) shows, the points on the graph represent the top 10 portfolios according to the fitness function. We can notice that there are some portfolios on the graph that meets the efficient frontier criteria which we are going to discuss in the results section.

VIII. Results Obtained Using the Hybrid Genetic Algorithm

Five different tests were applied. In each test, we selected different weights for the risk and return for portfolio fitness calculation.

a) *Risk & Return weights*

In this test we selected a weight of 0.5 for each of the risk and the return in the fitness function. Using these weights we obtained the following top 10 portfolios selected from the created 10,000 portfolios, the selection was based on the higher fitness.

Figure (4) shows the portfolios that are selected as the top 10 portfolios from the 10,000 portfolios we created based on the fitness function. The first six segments represent the chromosomes of the portfolio, where each chromosome represents a stock. The last two segments are the risk and the return respectively. Each chromosome is converted into the equivalent decimal number which represents the stock ID, and then the ID is used to calculate the stock weight in the portfolio.

```
Top 10 portfolios selected from the 10000 portfolios:
1010001 1111000 1000101 1101101 1110111 1100001  0.00771894662218939  0.094675255462184
1111000 1000101 1101101 1110111 1100001 1001011  0.00837236649310477  0.093257449915110
1100110 1010001 1111000 1000101 1101100 1110110  0.00624016282231191  0.091618289297658
1100111 1010001 1111000 1000101 1101100 1110110  0.00564187490618866  0.092039671118530
1010001 1111000 1000101 1101100 1110110 1100000  0.00599002959296613  0.091473947635135
1010001 1111000 1000101 1101100 1110110 1100001  0.00692759672921441  0.091847817875210
1100111 1010001 1111000 1000101 1101101 1110110  0.00561666820085502  0.092156281666666
1111000 1000101 1101100 1110110 1100000 1001011  0.00654011026950142  0.090016105802047
1111000 1000101 1101100 1110110 1100000 1001010  0.00638947117316662  0.090150601709401
1100111 1010001 1111000 1000101 1101101 1110111  0.00657791258693496  0.094838242928452
------------------------------------------------------------
```

For example let's consider the first portfolio shown in figure (4).

Figure 4: Top 10 Portfolios Selected from the 10,000 Portfolios

The portfolio chromosome is [1010001 1111000 1000101 1101101 1110111 1100001], this chromosome is composed of six different genes, when each gene is converted to its equivalent decimal number the portfolio stocks IDs are: [81 120 69 109 119 97]. Therefore, the weights for each stock are calculated as follows (Solimanpur, 2015):

$$v_1 = 1010001 \rightarrow v_1 = 81$$
$$v_2 = 1111000 \rightarrow v_2 = 120$$
$$v_3 = 1000101 \rightarrow v_3 = 69$$
$$v_4 = 1101101 \rightarrow v_4 = 109$$
$$v_5 = 1110111 \rightarrow v_5 = 119$$
$$v_6 = 1100001 \rightarrow v_6 = 97$$

The share of each company in this portfolio is obtained as follows (Solimanpur, 2015):

$$x_1 = \frac{81}{81+120+69+109+119+97} = 13.6\%$$
$$x_2 = \frac{120}{81+120+69+109+119+97} = 20.2\%$$
$$x_3 = \frac{69}{81+120+69+109+119+97} = 11.6\%$$
$$x_4 = \frac{109}{81+120+69+109+119+97} = 18.3\%$$
$$x_5 = \frac{119}{81+120+69+109+119+97} = 20.0\%$$
$$x_6 = \frac{97}{81+120+69+109+119+97} = 16.3\%$$

Then we use the top 10 portfolios to extract the genes which we are going to use in the crossover operation.

Figure (5) shows the genes and the stocks that were extracted from the top 10 portfolios. After extracting the genes we used them in the crossover operation to create the second generation of the portfolios, and then we selected the top 10 portfolios of the second generation based on the fitness function.

```
Top 10 portfolios stocks
131003 81      1010001
141004 120     1111000
141019 69      1000101
111004 109     1101101
141141 119     1110111
142041 97      1100001
141059 75      1001011
111001 102     1100110
141014 108     1101100
131062 118     1110110
141081 103     1100111
111033 96      1100000
141042 74      1001010
```

Figure 5: Genes and Stocks used to Create Top 10 Portfolios

Figure (6) shows the top 10 portfolios of the crossover results. These portfolios are used to draw the efficient frontier graph.

```
Crossover top 10 portfolios
1110111 1110110 1100110 1100000 1101101 1111000  0.00646079742249487  0.134291109939759
1110111 1110110 1100110 1001010 1101101 1111000  0.00675520227248665  0.129273693146417
1000101 1100110 1110111 1110110 1111000 1001010  0.007452091932551 8  0.118732436877076
1111000 1001011 1110111 1110110 1010001 1100000  0.00731863679208056  0.1211988472906 4
1100001 1110110 1111000 1001010 1110111 1000101  0.00817775246464712  0.119187514237856
1010001 1110111 1110110 1100110 1001010 1111000  0.00707252922642634  0.121227885993485
1110111 1110110 1001011 1100000 1101101 1111000  0.00695421613566113  0.1293090847723 7
1100110 1100000 1110110 1111000 1100001 1101101  0.00678583435607438  0.1055173504672 9
1100110 1100001 1110110 1111000 1001010 1110111  0.00787473981489722  0.126203511111111
1001010 1110111 1110110 1100110 1100000 1111000  0.00712075374560816  0.125906252782194
```

Figure 6: Top 10 Second Generation Portfolios

Figure (7) illustrates the generated efficient frontier by the proposed HGA. Overall, the annual return of optimal portfolios varies between 10.552% and13.429%, while their risk changes between 0.646% and 0.818%. A decision-making technique based on AHP is proposed in the next section which helps decision-makers to select the most suitable portfolio from among 10 portfolios.

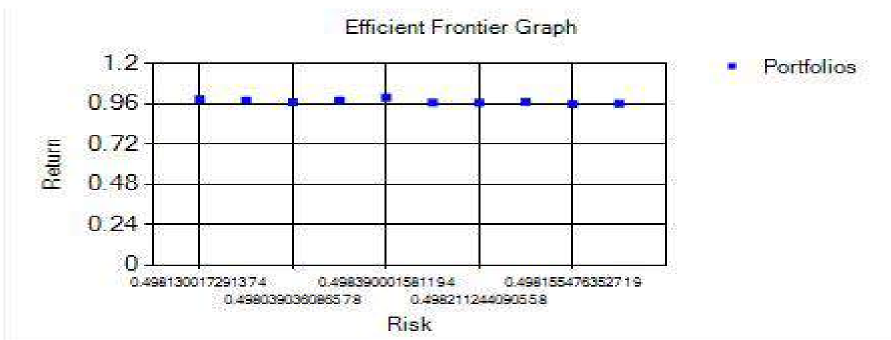

Figure 7: Efficient Frontier Graph

b) Portfolio Selection via Analytic Hierarchy Process

The proposed hierarchical structure of the optimum portfolio selection problem along with the alternatives obtained by HGA and the identified criteria are depicted in Figure (8). As seen, the decision hierarchy consists of three levels. The optimum portfolio selection is the prime objective of the problem and takes place at the topmost level (Level 1) of the hierarchy. The seven criteria: return, risk, beta ratio, liquidity, RVAR, TR and alpha ratio take place at the second level. Finally, the 10 portfolios identified by HGA take place at the most bottom level (Level 3) as decision alternatives.

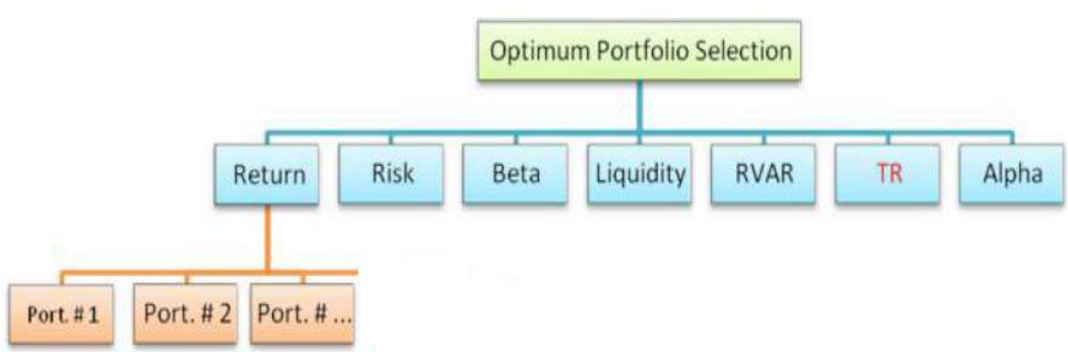

Figure 8: Proposed Hierarchy of the Portfolio Selection Problem

Table (2) shows the top 10 portfolios that resulted from the (GA) operation and their criteria which were mentioned before, and it should be noted that the Sharpe and Tryner for the market portfolio was calculated and they were (-8.646) and (-0.038049125) respectively.

Table 2: Seven Criteria's Portfolios

Por.#	Risk	Return	liquidity	Sharpe	Beta	TR	Alpha
1	0.646%	13.429%	1.702	14.904	0.153	0.631	0.102
2	0.676%	12.927%	1.815	13.512	0.191	0.478	0.099
3	0.745%	11.873%	2.159	10.834	0.197	0.410	0.088
4	0.732%	12.120%	2.462	11.368	0.295	0.282	0.094
5	0.818%	11.919%	2.301	9.928	0.146	0.554	0.087
6	0.707%	12.123%	2.018	11.768	0.281	0.296	0.094
7	0.695%	12.931%	1.913	13.130	0.211	0.434	0.099
8	0.679%	10.552%	0.860	9.950	0.133	0.507	0.073
9	0.787%	12.620%	1.966	11.201	0.335	0.263	0.101
10	0.712%	12.591%	1.843	12.345	0.166	0.529	0.094

Figure (9), shows the result of the hierarchy analysis process, where the (Y) axis represents the portfolio number and the (X) axis represents the relative weight of the portfolio in terms of preference. As illustrated by the figure the optimum portfolio is portfolio number (8).

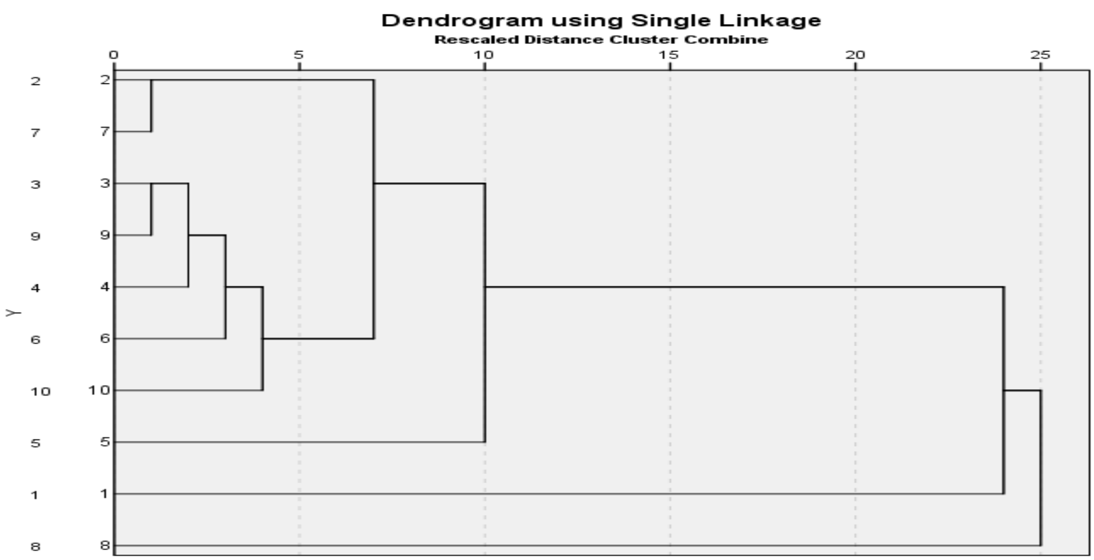

Figure 9: Portfolio Hierarchy Analysis

Therefore, the portfolios can be displayed according to its optimality as in the following table (3):

Table 3: Portfolios Optimality

Portfolio #	Optimality Order
1	2
2	7
3	7
4	6
5	3
6	5
7	7
8	1
9	7
10	4

IX. Conclusion and Policy Implications

The main objective of this study is to investigate the ability of a Hybrid Genetic Algorithm and Analytic Hierarchy Process in selecting the optimum portfolio. The study creates portfolios using the daily return of the companies listed in Amman Stock Exchange, during the period from January 1, 2015 to December 31, 2015. With reference to the findings of the analysis, the findings could be listed as follows:

1) Hybrid Genetic Algorithm can identify the portfolios on the efficient frontier.
2) Hybrid Genetic Algorithm does not have any restriction as far as the number of assets, is concerned.
3) Hybrid Genetic Algorithms have advantage over problems for the portfolio selection cases which scale of the problem or the nonlinear constraints of the problem unable us to use linear or quadratic models.
4) Analytic Hierarchy Process can select the optimum portfolio among the portfolios obtained by HGA.
5) Finally, the selected optimal portfolio achieves a return of 10.552% with a risk of 0.679%, where the liquidity of this portfolio is 0.860, Sharpe ratio 9.950, Beta 0.133, Try nor ratio 0.507 and Alpha ratio 0.073.

a) *Policy implications*

The results discussed above can lend support the following recommendations:

1) To make portfolio selection and optimization problems more accurate, we can present other main parameters such as taxes and transaction costs in order to make more perfect decisions.
2) AHP in this article includes seven criteria. This hierarchy can be more complete by adding other quantitative or qualitative criteria not used here.
3) Individuals, Investors and governments can employ this method to construct optimized portfolio and modify their portfolios investment depending on their Investment strategy.

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