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I. Introduction

In many manufacturing firms where there exists mass production, measurement made on each product is subject to error due to variation from one item to the other. Since there must be variations, it becomes important to study and determine when any observed variation is significant or not. This is the reason why the Federal Government of Nigeria came up with legislations to protect the buyers from buying inferior goods. Increase in consumer buying behaviou towards some selected drinks will directly affect the production of such drinks in our breweries industry. Quality control relies partly upon patronage and some other reliable factors, in beer production process, the measurement of attributes such as fill height and level of co_2 is of paramount important and that is the reason why quality control is evolving in developing systems to ensure standard products or services as well as meeting or exceeding customer's requirements. Walter Shewhart introduced the concept of statistical quality control thereby controlling quality of mass produced goods. Shewhart believed that variation always exists in manufactured products and that the variation can be studied, monitored and controlled using Statistics. Walter Shew hart explained the theories about using statistical quality control charts to improve quality and productivity in which case he developed fourte en points agenda for companies to improve quality and productivity, reduce costs and compete effectively in the world market.

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II. Literature Review

Reeves and Bednar (1994) define quality as excellence, value, conformance to specifications, and meeting or exceeding customers' expectation. The term "fitness for use" defined by Juran (1974) is also included in the quality definition presented by Reeves and Bednar (1994). Thus, the customer perspective with respect to quality is the master key that should be understood while determining any term for quality or definition of quality. Deming, W.E (1986). worked on Quality and Productivity Improvement using acceptance sampling method, and he was able to obtain increase in quality and simultaneous reduction in the cost of reducing waste, re write staff attrition and litigation while increasing customer's loyalty. Farhat, B. A. and Al-Darrab, I. (1998). Total quality management is now established and widely used management process. One of its associated features is the application of statistical quality control techniques. A quality product or service is one that meets the customer's needs and provides the value that they want and expect. They are also of the opinion that quality management is a formal approach to management in which the overriding priority of the organization is to deliver a quality product or service and to work towards excellence and continuous improvement in everything it does.

Quality can be viewed from the perspectives of design and product in which case; design quality is the different grades or levels of performance, reliability, serviceability and function that are the results of deliberate engineering and management decision. On the other hand, product quality is the conformance of the product with specifications or expectations of the user in terms of fitness for use and cost. They are also of the opinion that control charts are closely related with statistical test of hypothesis. The control chart is a test of hypothesis that the process is in a state of statistical control. Shres tha and Chalidabhongse (2006) explained over their survey on 300 employees working in 60 Thai companies to what extent job satisfaction is affected by the existing performance appraisal system used by these companies. They concluded that since the performance appraisal system is part of the company's running processes, employees would show lower performance level if the appraisal system is not satisfactorily controlled.

Cooper (2008) have emphasized on the impact of TQM practices on job satisfactions. The main aim of their research was to examine the relationship between people-related TQM practices and job satisfaction of service employees. The study triggers the question whether a TQM has an effect on employees' satisfaction. Pitterman (2000)'s findings on Telecordia technologies showed that customer satisfaction figures had gone up from 60% in 1992 to 95% at the time of implementing ISO 9001 quality system. Also, there was a 63% reduction noted in test cost efficiency since 1993 that 98% of major software released by Telcordia between 1995 and 1998 were delivered in time, even though the number of releases had tripled during the four year time.

Takala et al. (2006) have gone even further to seeking customer satisfaction by improving and ensuring that customer satisfaction survey is supposed to be well designed and validated in order to be an effective measurement tool for its intended purpose. In their research paper, the purpose was to verify the reliability of customer satisfaction survey in context to three aspects of service; quality, delivery and responsiveness. They concluded that there was a need to work on the flexibility of the customer satisfaction survey to ensure the reliability in the qualitative analysis of the supply chain. Sitko-Lutek et al. (2010) examined the customer complaint handling process with respect to the information quality, thereby suggesting possible areas of improvements in the process. Their research method involved reviewing documents, complaint handling procedures and interviews through a social network analysis (SNA) model. The software used for SNA was UCI net and the results suggested that process engineering leadership played a vital and responsive role in disseminating quality assurance information in identifying potential areas of process improvements, thereby enhance and improve the company's profit and customers satisfaction.

III. Methodology

A control chart is a graphical representation that shows whether a sample data falls within a normal range of variation. It used to know if a process is in statistical quality control or not. It is also a graphical representation of mathematical model used to monitor a process in order to detect changes in parameter of that process. It displays the quality characteristics that has been measured or computed from a sample against the sample number or time. They are simple to construct and to interpret as they employ a center line (denoted as CNL) and two major control limits; an upper control limit (denoted as UCL) and a lower control limit (denoted as LCL). The center line represents the average performance of the process when it is in a state of statistical control- that is, when only common cause variation exists. The upper and lower control limits are horizontal lines situated above and below the center line. These control limits are established so that when the process is in control, almost all plots will be between the upper and lower limits.

In practice,

- If all observed plot points are between the LCL and UCL and if no unusual pattern of points exists, we have no evidence that assignable causes exist and we assume that the process is in statistical control. In this case, only common causes of the process variation exist, and no action to remove assignable causes is taken on the process. If we were to take such action, we would be unnecessarily tempering with the process.
- If we observe one or more plot points outside the control limits, then we have evidence that the process is out of control due to one or more assignable causes. Here we must take action on the process to remove those assignable causes.

a) *Multivariate Quality Control Chart*

Multivariate methods that consider the variables jointly are required. Process-monitoring problems in which several related variables are of interest are sometimes called *multivariate quality-control (or process-monitoring) problems.* The original work in multivariate quality control was done by Hotelling (1947), who applied his procedures to bombsight data during World War II. Subsequent research dealing with control procedures for several related variables include Hicks (1955), Jackson (1956, 1959, 1985), Crosier (1988), Hawkins (1991, 1993b), Lowry et al. (1992), Lowry and Montgomery (1995), Pignatiello and Runger (1990), Tracy, Young, and Mason (1992), Montgomery and Wadsworth (1972), and Alt (1985). This subject is particularly important today, as automatic inspection procedures make it relatively easy to measure many parameters on each unit of product manufactured.

The HotellingT^2 chart is the analog of the Shewhartx chart. Multivariate control charts work well when the number of process variables is not too large—say, 10 or fewer. As the number of variables grows, however, traditional multivariate control charts lose efficiency with regard to shift detection. A multivariate approach should be used to monitor process stability with more than one important characteristic. This approach can account for correlations between characteristics and will control the overall probability of falsely signaling a special cause of variation when one is not present. The most common multivariate chart is the T^2 chart. There are many situations in which the simultaneous monitoring or control of two or more control of two or more related quality characteristics is necessary. The process is considered to be in control only if the sample means x_1 and x_2 fall within their respective control limits. Monitoring these two quality characteristics independently can be very misleading. So it is best we use the HotellingT^2 control chart.

b) The Multivariate Normal Distribution

In univariate statistical quality control, we generally use the Normal distribution to describe the behaviour of a continuous quality characteristic. The Univariate Normal probability density function is

f(x)=1/(√2πσ^2) e^(-1/2 〖((x- μ)/σ)〗^2) - ∞<x < ∞ … (1)

The mean of the normal distribution is μ and the variance is σ2. Note that (apart from the minus sign) the term in the exponent of the normal distribution can be written as follows:

(x- μ) (σ^2)^(-1) (x- μ) (2)

This quantity measures the squared standardized distance from x to the mean, where by the term "standardized" we mean that the distance is expressed in standard deviation units. This same approach can be used in the multivariate normal distribution case. Suppose that we have p variables, given by x1, x2, . . . ,xp. Arrange these variables in a p-component vector x′ = [x1, x2, . . . ,xp]. Let μ1 = [μ, μ2, . . . , μp] be the vector of the means of the x's, and let the variances and covariances of the random variables in x be contained in a p * p covariance matrix Σ

The main diagonal elements of Σare the variances of the x's and the off-diagonal elements are the covariances. Now the squared standardized (generalized) distance from x to μis

$$(x-\mu)'\Sigma^{-1}(x-\mu) \quad (3)$$

The multivariate normal density function is obtained simply by replacing the standardized distance in equation (.2) by the multivariate generalized distance in equation (3) and changing the constant term to a more general form that makes the area under the probability density function unity regardless of the value of *p*. Therefore, the *multivariate normal* probability density function is

$$f(x)=\frac{1}{(2\pi)^{p/2}|\Sigma|^{1/2}}e^{-\frac{1}{2}(x-\mu)'\Sigma^{-1}(x-\mu)} \quad (4)$$

where $-\infty < x_j < \infty, j = 1, 2, . . . , p.$

A multivariate normal distribution for $p = 2$ variables (called a bivariate normal).

$$f(x)=\frac{1}{2\pi|\Sigma|^{1/2}}e^{-\frac{1}{2}(x-\mu)'\Sigma^{-1}(x-\mu)}\ldots \quad (5)$$

c) The Sample Mean Vector and Covariance Matrix

Suppose that we have a random sample from a multivariate normal distribution—say,

where the *I* th sample vector contains observations on each of the *p* variables $xi_1, xi_2, . . . , xi_p$.
Then the sample mean vector is

$$\bar{x}_j \quad = \frac{1}{n}\sum_{i=1}^{n} x_{ij} \; \{j=1,2,\ldots,p\} \quad (6)$$

and the sample variance is

$$s_j^2 \quad = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_{ij} \; - \bar{x}_j\right)^2 \{j=1,2,\ldots,p\} \quad = p\} \quad (7)$$

and the sample covariance is

$$s_{jhk} = \frac{1}{n-1}\sum_{i=1}^{n}(x_{ijk} \; - \bar{x}_{jk})\,(x_{ihk} \; - \bar{x}_{hk})\begin{cases} k \; = \; 1,2,\ldots,m \\ \quad j \; \neq \; h \end{cases} \quad (8)$$

d) Hotelling T^2 Control Chart

It is the most familiar multivariate process-monitoring and control procedure. Hotelling T^2 control chart is for monitoring the mean vector of the process. It is a direct analog of the univariate Shewhart chart. There are two versions of the Hotelling T^2 charts which are Sub grouped data and Individual observations.

e) Subgrouped Data

Suppose that p quality characteristics x_1, x_2, ...,x_p are jointly distributed according to the multivariate normal distribution (see equation 3.6.4). Let μ_1, μ_2, ...,μ_p be the mean values of the quality characteristics and let σ_{jk}'s represent the variance-covariance values of the p-characteristics. In practice, it is usually necessary to estimate Σ and μ from the preliminary samples of size n, taken when the process is assumed to be in control. Suppose that m such samples are available. the sample means and variances are calculated from each sample as usual; that is,

$$\bar{x}_{jk} = \frac{1}{n}\sum_{i=1}^{n} x_{ijk} \begin{cases} j = 1,2,\ldots,p \\ k = 1,2,\ldots,m \end{cases} \tag{9}$$

$$s_{jk}^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_{ijk} - \bar{x}_{jk}\right)^2 \begin{cases} j = 1,2,\ldots,p \\ k = 1,2,\ldots,m \end{cases} \tag{10}$$

where x_{ijk} is the *I* th observation on the *j* th quality characteristics in the *k*th sample. The covariance between quality characteristic *j* and quality characteristic *h* in the *k* th sample is

$$s_{jhk} = \frac{1}{n-1}\sum_{i=1}^{n}(x_{ijk} - \bar{x}_{jk})(x_{ihk} - \bar{x}_{hk}) \begin{cases} k = 1,2,\ldots,m \\ j \neq h \end{cases} \tag{11}$$

The statistics $\bar{x}_{jk}$, s_{jk}^2 and s_{jhk} are then averaged over all m samples to obtain

$$\bar{\bar{x}}_j = \frac{1}{m}\sum_{k=1}^{m} \bar{x}_j \ldots\ldots\ldots\ldots\ldots (3.6.12)\ \bar{s}_j^2 = \frac{1}{m}\sum_{k=1}^{m} s_{jk}^2 \tag{12}$$

and

$$\bar{s}_{jh} = \frac{1}{m}\sum_{k=1}^{m} s_{jhk} \quad (13)$$

The $\{\bar{\bar{\boldsymbol{x}}}_j\}$ are the elements of the vector $\bar{\bar{\boldsymbol{x}}}$, and the *p* x *p* average of sample covariance matrices S is formed as

$$S = \begin{bmatrix} \bar{s}_1^2 & \cdots & \bar{s}_{1p} \\ \vdots & \ddots & \vdots \\ \bar{s}_{p1} & \cdots & \bar{s}_p^2 \end{bmatrix} \tag{15}$$

To use the T^2 Control Chart, we will use the test statistics;

$$T^2 = n(\bar{x} - \bar{\bar{x}})' S^{-1}(\bar{x} - \bar{\bar{x}})$$

Control chart. This is a directionally invariant control chart; that is, its ability to detect a shift in the mean vector only depends on the magnitude of the shift, and not in its direction. There are two distinct phases of control chart usage.

PHASE I is the use of the charts for establishing control; that is, testing whether the process is in control when the *m* preliminary subgroups are drawn. The control limit for T^2 control chart are given by

$$UCL = \frac{p(m-1)(n-1)}{mn-m-p+1} F_{\propto,p,mn-m-p+1} \quad and\ LCL = \quad 0 \tag{17}$$

Phase II is the use of the chart for monitoring future production, sample size of at least n= 200 is needed. The control limits are as follows:

$$UCL = \frac{p(m+1)(n-1)}{mn-m-p+1} F_{\propto,p,mn-m-p+1} \quad and\ LCL = \quad 0 \tag{18}$$

f) Individual Observation

Here, multivariate control charts with subgroup size, *n* = *1* is of interest. Suppose that *m* samples, each of size *n* = *1*, are available and that *p* is the number of quality characteristics observed in each sample. Let $\bar{\boldsymbol{x}}$ and S be the sample mean vector and covariance matrix, respectively, of these observations. The Hotelling T^2 statistic in equation becomes

$$T^2 = (x - \bar{x})' S^{-1} (x - \bar{x}) \quad (19)$$

The phase II control limits for this statistic are

$$UCL = \frac{p(m+1)(m-1)}{m^2 - mp} F_{\propto,p,m-p} \quad and \quad LCL = \quad (20)$$

When the number of preliminary samples *m* is large, say m > 100, most practitioners use an approximate control limit, either

$$UCL = \frac{p(m-1)}{m-p} F_{\propto,p,m-p} \quad (21)$$

$$UCL = \mathcal{X}^2_{\propto,p} \quad (22)$$

However, for m> 100, equation (21) is a reasonable approximation.
For phase i, the limits are based on a beta distribution,

$$UCL = \frac{(m-1)^2}{m} \beta_{\propto,\frac{p}{2},\frac{m-p-1}{2}} \quad and \quad LCL = 0 \quad (23)$$

Where $\beta_{\propto,p/2,(m-p-1)/2}$ is the upper α percentage point of a beta distribution with parameters p/2 and (m-p-1)/2. Approximations to the phase I limit based on the F and chi-square distributions are likely to be inaccurate. Basically, the focus will be on the Sub grouped data because it suits the type of data that was collected.

g) Control Chart for Monitoring Variability

Monitoring multivariate process are in two levels, which are to monitor the process mean vector *m* and to monitor process variability. Process variability is summarized by the *p* x *p* covariance matrix Σ. The main diagonal elements of this matrix are the variances of the individual process variables, and the off-diagonal elements are the covariances. We can use the approach based on the sample *generalized* variance, |S|. This statistic, which is the determinant of the sample covariance matrix, is a widely used measure of multivariate dispersion. Another method would be to use the mean and variance of |S|, that is, *E*(|S|) and *V*(|S|), and the property that most of the probability distribution of |S| is contained in the interval

$$E(|S|) \pm 3\sqrt{(V(|S|))}.$$

It can be shown that

$$E(|S|) = b_1|\Sigma| \quad and \quad V(|S|) = b_2|\Sigma|^2 (24)$$

where

$$b_1 = \frac{1}{(n-1)^p} \prod_{i=1}^{p} (n-i) \quad (25)$$

and

$$b_2 = \frac{1}{(n-1)^{2p}} \prod_{i=1}^{p} (n-i) \left[\prod_{j=1}^{p} (n-j+2) - \prod_{j=1}^{p} (n-j) \right] \quad (26)$$

Therefore, the parameters of the control charts for |S| would be

$$UCL = |\Sigma| \left(b_1 + 3b_2^{\frac{1}{2}} \right)$$

$$CL = b_1|\Sigma|$$

$$LCL = |\Sigma| (b_1 + 3b_2^{1/2}) (27)$$

The lower control limit in equation (27) is replaced with zero if the calculated value is less than zero. In practice, Σ usually will be estimated by a sample covariance matrix S, based on the analysis of preliminary samples. If this is the case, we should replace $|\Sigma|$ in equation (27) by $|S|/b_1$

In this study, two measurement quality characteristics are being analyzed using Multivariate statistical quality control.

Fill height: It measures the level of liquid in a bottle of drink. The products under study are STAR, MALTINA and GOLDBERG from Nigerian Breweries plc. The standard is always at 60cl.

Co_2 level: It measures the level of co_2 in each bottle. The target for corking a bottle of STAR is between (0.52-0.54%wt/wt), that of MALTINA is (0.59-0.61%wt/wt) and GOLDBERG is (0.62- 0.64%wt/wt) where %wt/wt means weight per weight.

Data Presentation: The data used for this analysis is shown in the appendix ‘A to appendix F.

IV. Data Analysis and Results

In this chapter, the Hotelling T^2 control chart is used for the analysis of fill height and level of co_2 measurements of Star, Maltina and Goldberg using R.

Analysis on the Fill Height Measurement and co_2 Level of Star

The fill height of STAR refers to the height of the liquid content in a bottle of a STAR. And the co_2 level refers to the level of co_2 in each bottle of STAR. There can be cases of low fill, high fill and normal fill. The normal or standard fill height of STAR of the company is 60cl. And the standard co_2 level of STAR is between (0.52-0.54%wt/wt). The tables below display analysis carried out using R on various readings on fill height and co_2 level that was observed at different times.

Table 1: Star

	Means		Variance and Covariances			Control Chart Statistics	
Sample Number k	Fill Height ($\bar{x}_{1k}$)	Level of CO_2 ($\bar{x}_{2k}$)	S^2_{1k}	S^2_{2k}	S_{12k}	Hotelling T^2_k	$\|S_k\|$
1	60.0	0.518	1.5	0.00037	0.0200	3.8339370	0.00015500
2	59.6	0.530	1.3	0.00035	0.0100	0.1962245	0.00035500
3	59.8	0.520	0.7	0.00005	0.0000	2.6772350	0.00003500
4	60.0	0.530	1.0	0.00010	0.0025	0.1181133	0.00009375
5	59.8	0.546	2.2	0.00013	0.0015	7.7073474	0.00028375
6	60.0	0.526	1.0	0.00013	-0.0025	0.4357597	0.00012375
7	59.4	0.51		0.00	-0.0005	6.45326	0.00010375
8	60.0	0.522	1.5	0.00007	0.0075	1.6743676	0.00004875
9	60.0	0.548		0.00	0.0000	10.0856	0.00030000
10	59.4	0.528	0.3	0.00017	-0.0065	0.8187689	0.00000875
1	60.0	0.51		0.00	-0.0025	6.914467	0.00012375
12	59.6	0.534	0.8	0.00003	-0.0030	0.6801556	0.00001500
13	60.0	0.54		0.00	0.0000	10.0856	0.00025500
14	59.8	0.536	2.7	0.00053	-0.0285	1.1678815	0.00061875
15	60.0	0.516		0.00	-0.0125	5.259082	0.00007375
16	59.6	0.550	0.3	0.00005	-0.0025	11.8254949	0.00000875
17	60.0	0.52		0.00	0.0025	0.93994	0.00006875
18	59.8	0.534	1.7	0.00008	-0.0015	0.5507094	0.00013375
19	59.8	0.52		0.00	0.0035	0.91845	0.00007875
20	60.0	0.528	1.5	0.00057	-0.0225	0.1618163	0.00034875
Averages	59.8	0.5296		0.00	-0.0225		

Table1.0 shows the Variances and Covariances of the fill height and level of CO_2 of STAR and also the Hotelling T^2 and Variability of each of the 20 samples. The Grand mean, Variance-Covariance Matrix (s) for the control limit used in the Variability plot, and the control Limits for the Hotelling T^2 and Variability plot are represented in the table(s) below.

Grand Mean

Fill Height	59.8300
Level of CO_2	0.5296

Variance-Covariance Matrix (S) for the control limit used in the variability plot

	Fill Height	Level of CO_2
Fill Height	1.5000	-0.02250
Level of CO_2	-0.0225	0.00057

Control Limits for the Hotelling T^2 and Variability plot

Variability	0	0.0002345375	0.001096283
Hotelling T^2	0	-	14.523838130

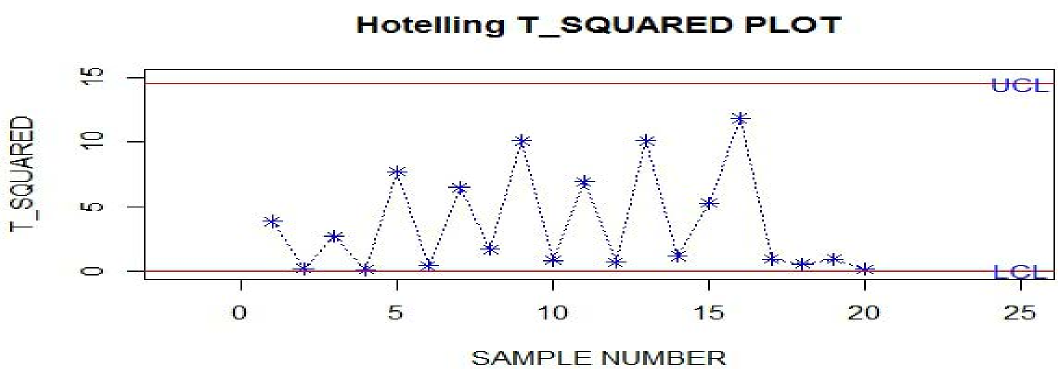


Figure 1: HotellingT-Square plot of Star

h) Interpretation of Star Chart

From the Variability plot above, most of the sample variances are on or close to the lower control limit (LCL) while they are very far from the upper control limit, which means that the variability (the variances of the observation from the mean) is in control. Thus, the Hotelling T^2 can be plotted to see if the process is actually in control. From the Hotelling T^2 plotted above also, it can be seen that all the plot point fall within the UCL and LCL, which means that it can be concluded that the fill height and level of Co_2 of STAR is under control. The R code was used for the analysis of STAR.

Analysis on the Fill Height Measurement and co_2 Level of Maltina.

The fill height of MALTINA refers to the height of the liquid content in a bottle of a MALTINA. And the co_2 level refers to the level of co_2 in each bottle of MALTINA. There can be cases of low fill, high fill and normal fill. The normal or standard fill height of MALTINA of the company is 60cl. And the standard co_2 level of MALTINA is between (0.59-0.61%wt/wt). The tables below display analysis carried out using R on various readings on fill height and co_2 level that was observed at different times.

Table 2: Maltina

Means	Variances and Covariances			Control Chart Statistics			
Sample Number k	Fill Height ($\bar{x}_{1k}$)	Level of CO_2 ($\bar{x}_{2k}$)	S^2_{1k}	S^2_{2k}	S_{12k}	Hotelling T^2_k	$\lvert S_k \rvert$
1	60.0	0.596	1.0	0.00003	0.0025	1.7606748	0.00002375
2	59.6	0.590	1.3	0.00010	0.0100	8.0208486	0.00003000
3	59.8	0.598	1.2	0.00007	0.0070	0.3296529	0.00003500
4	60.0	0.598	0.5	0.00017	0.0075	0.6161894	0.00002875
5	59.6	0.598	1.3	0.00002	-0.0010	0.4235547	0.00002500
6	60.0	0.594	0.5	0.00003	0.0025	3.5875338	0.00000875
7	59.4	0.602	1.3	0.00002	-0.0035	1.4622119	0.00001375
8	60.0	0.598	0.0	0.00007	0.0000	0.6161894	0.00000000

9	60.0	0.598	3.5	0.00007	0.0125	0.6161894	0.00008875
10	59.4	0.602	1.3	0.00002	-0.0010	1.4622119	0.00002500
11	60.0	0.608	1.5	0.00007	0.0075	5.1293660	0.00004875
12	59.6	0.608	0.3	0.00002	-0.0010	6.2803431	0.00000500
13	60.0	0.608	1.0	0.00007	0.0050	5.1293660	0.00004500
14	59.8	0.610	0.7	0.00010	-0.0050	8.5987526	0.00004500
15	60.0	0.604	1.5	0.00003	0.0025	1.2769746	0.00003875
16	59.6	0.602	1.3	0.00007	0.0010	0.7191493	0.00009000
17	60.0	0.602	0.5	0.00002	0.0000	0.3743393	0.00001000
18	59.8	0.596	0.7	0.00008	0.0065	1.3397771	0.00001375
19	59.8	0.596	1.7	0.00013	-0.0085	1.3397771	0.00014875
20	60.0	0.592	1.5	0.00007	0.0000	6.0967665	0.00010500

Table2 shows the Variances and Covariances of the fill height and level of Co_2 of MALTINA and also the Hotelling T^2 and Variability of each of the 20 samples. The Grand mean, Variance-Covariance Matrix (s) for the control limit used in the Variability plot, and the control Limits for the Hotelling T^2 and Variability plot are represented in the table(s) below.

Grand Mean

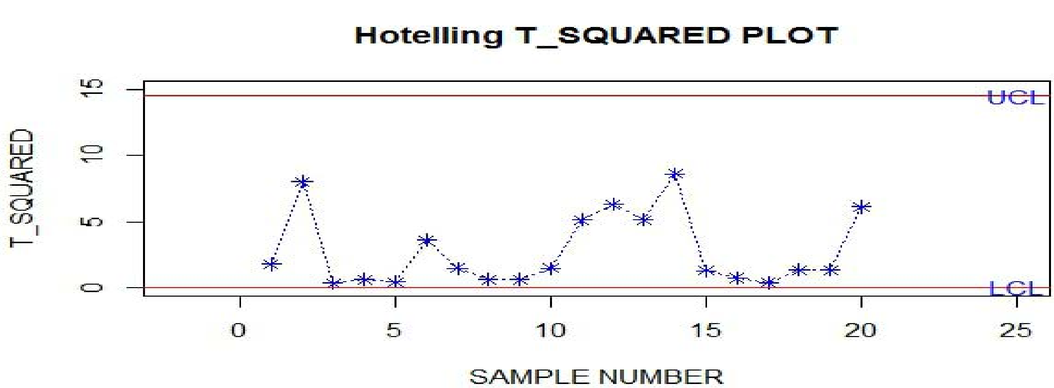
Fill Height	59.82
Level of CO_2	0.60

Variance-Covariance Matrix (S) for the control limit used in the variability plot

	Fill Height	Level of CO_2
Fill Height	1.130000	0.002225
Level of CO_2	0.002225	0.000063

Control Limits for the Hotelling T^2 and Variability plot

	LCL	CL	UCL
Hotelling T^2	0	-	14.52384
Variability	0	0.00006624	0.001096



Interpretation of Maltina Chart

Figure 2: HotellingT-Square plot of Maltina

From the Hotelling T^2 plotted above, it can be seen that all the plot point fall within the UCL and LCL, which means that the fill height and level of co_2 of MALTINA is under control.

Analysis on the Fill Height Measurement and Co_2 Level oif Goldberg

The fill height of GOLDBERG refers to the height of the liquid content in a bottle of a GOLDBERG. And the co_2 level refers to the level of co_2 in each bottle of GOLDBERG. There can be cases of low fill, high fill and normal fill. The normal or standard fill height of GOLDBERG of the company is 60cl. And the standard level of co_2GOLDBERG is between (0.62- 0.64%wt/wt). The tables below display analysis carried out using R on various readings on fill height and co_2 level that was observed at different times.

Table 3: GOLDBERG

	Means		Variances and Covariances			Control Chart Statistics	
Sample Number k	Fill Height ($\bar{x}_{1k}$)	Level of CO_2 ($\bar{x}_{2k}$)	S^2_{1k}	S^2_{2k}	S_{12k}	Hotelling T^2_k	$\lvert S_k \rvert$
1	60.0	0.642	1.0	0.00007	0.0075000	2.28910604	0.00001375
2	59.6	0.636	1.3	0.00008	0.0030000	0.28786007	0.00009500
3	59.8	0.630	0.7	0.00025	-0.0025000	1.31283293	0.00016875
4	60.0	0.634	0.5	0.00008	0.0025000	0.25264057	0.00003375
5	59.6	0.626	0.3	0.00013	-0.0045000	4.15963167	0.00001875
6	60.0	0.630	1.0	0.00010	-0.0075000	1.58110868	0.00004375
7	59.4	0.638	0.8	0.00007	0.0060000	1.37548756	0.00002000
8	60.0	0.632	0.5	0.00037	0.0010000	0.72131622	0.00008500
9	60.0	0.646	3.5	0.00008	0.0000000	5.65403962	0.00028000
10	59.4	0.634	1.3	0.00003	-0.0045000	0.88317403	0.00001875
11	60.0	0.644	1.5	0.00008	-0.0025000	3.77601443	0.00011375
12	59.6	0.632	0.3	0.00007	0.0010000	0.66321829	0.00002000
13	60.0	0.632	0.5	0.00002	0.0000000	0.72131622	0.00001000
14	59.8	0.646	2.7	0.00008	0.0015000	5.72751642	0.00021375
15	60.0	0.646	1.0	0.00008	0.0050000	5.65403962	0.00005500
16	59.6	0.628	1.3	0.00007	0.0040000	2.60304373	0.00007500
17	60.0	0.634	0.5	0.00008	0.0000000	0.25264057	0.00004000
18	59.8	0.630	0.7	0.00010	-1.1×10^{-22}	1.31283293	0.00007000
19	59.8	0.634	0.7	0.00018	0.0010000	0.06980296	0.00012500
20	60.0	0.630	0.5	0.00005	0.00250000	1.58110868	0.00001875

Table 3 shows the Variances and Covariances of the fill height and level of Co_2 of GOLDBERG and also the Hotelling T^2 and Variability of each of the 20 samples.

The Grand mean, Variance-Covariance Matrix (s) for the control limit used in theVariability plot, and the control Limits for the Hotelling T^2 and Variability plot are represented in the table(s) below.

Grand Mean

Fill Height	59.820
Level of CO_2	0.6352

Variance-Covariance Matrix (S) for the control limit used in the variability plot

	Fill Height	Level of CO_2
Fill Height	1.0300000	0.0011250
Level of CO_2	0.0011250	0.0001035

Control Limits for the Hotelling T^2 and Variability plot

	LCL	CL	UCL
Hotelling T^2	0	-	14.52384
Variability	0	0.0001053	0.000492

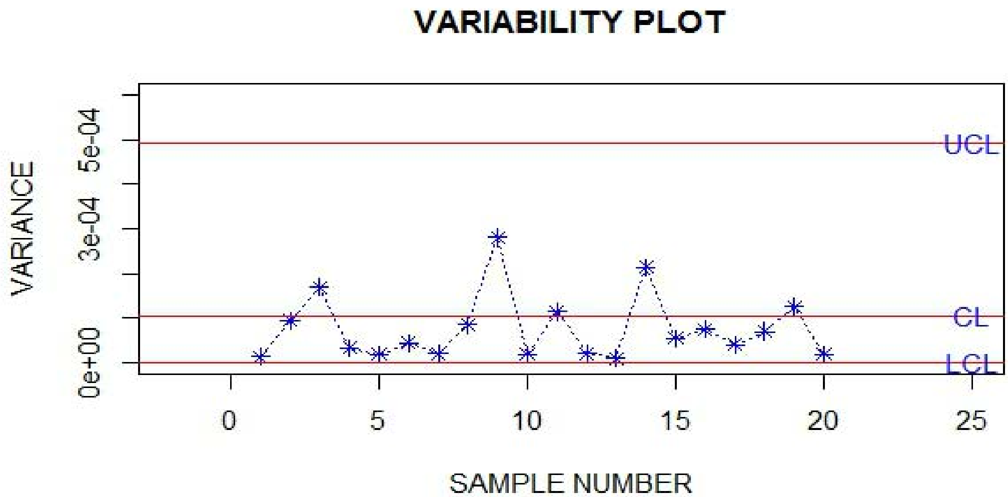


Figure 3: Variability plot of Goldberg

From the Variability plot above, most of the sample variances are on or close to the lower control limit (LCL) while they are very far from the upper control limit, which means that the variability (the variances of the observation from the mean) is in control.

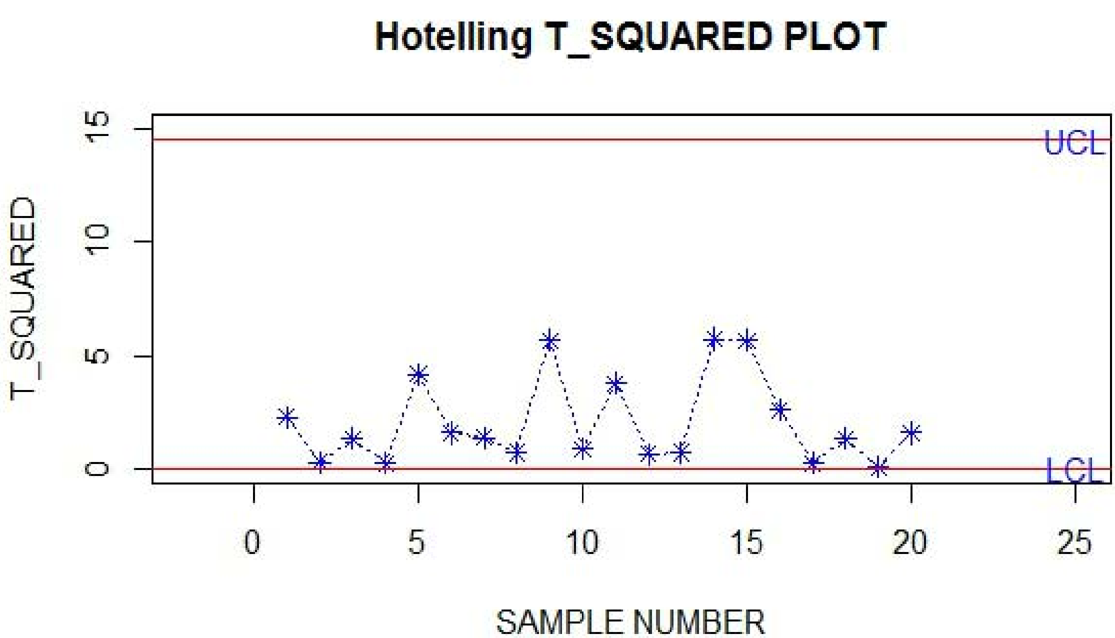


Figure 4: HotellingT-Square plot of Goldberg

From the Hotelling T2 plotted above also, it shows that all the plot point fall within the UCL and LCL, which means that the fill height and level of Co_2 of GOLDBERG is under control.

Based on the results obtained from the analysis so far for all the drinks considered, none of the characteristics examined and analyzed fall within the control which invariably means we do not have sufficient evidence to reject the null hypothesis hence we Accept the null hypothesis for both the fill height and the level of Co2.

V. Conclusion

The results obtained from the method used show that the components for the production of the beer under consideration(fill height and level of Co2) shows that the variability of the three products are in control, and this information helped in proceeding to check if the two quality characteristics are in control, also, using the Hotelling T2 control chart of Sub grouped data, the values were all within the lower and upper control limit for the three products, which helps to affirm the fact that the quality characteristics of STAR, MALTINA AND GOLDBERG are in control. This shows that the Quality Control Unit of the Company should not relent in carrying out their test on the products, all these will help the company to maintain the required standard and survive competition with other likely products from other company.

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Appendix

Table A: Showing the data of the fill height measurement of star in cl

Sample No	Time	A	B	C	D	E
1	7:00am	60	59	60	62	59
2	8:00am	59	58	60	60	61

3	9:00am	59	60	60	61	59
4	10:00am	60	61	61	59	59
5	11:00am	60	59	60	58	62
6	12 noon	61	59	60	59	61
7	1:00pm	60	59	59	58	61
8	2:00pm	62	60	59	60	59
9	3:00pm	58	62	60	59	61
10	4:00pm	60	59	59	60	59
11	5:00pm	61	59	61	60	59
12	6:00pm	60	59	59	61	59
13	7:00pm	58	60	60	61	61
14	8:00pm	57	60	61	61	60
15	9:00pm	59	61	59	60	61
16	10:00pm	60	59	60	60	59
17	11:00pm	58	60	62	59	61
18	12:00am	59	60	59	62	59
19	1:00am	59	60	59	60	61
20	2:00am	60	60	59	59	62

Note: A, B, C, D, and E are the numbers of observations for each samples respectively.

Table B: Showing the data of the level of co_2 in each bottle of star in wt/wt

Sample No	Time	A	B	C	D	E
1	7:00am	0.52	0.51	0.50	0.55	0.51
2	8:00am	0.55	0.50	0.54	0.53	0.53
3	9:00am	0.53	0.52	0.52	0.52	0.51
4	10:00am	0.54	0.54	0.52	0.52	0.53
5	11:00am	0.53	0.56	0.55	0.54	0.55
6	12noon	0.51	0.52	0.54	0.53	0.53
7	1:00pm	0.52	0.51	0.53	0.51	0.51
8	2:00pm	0.53	0.52	0.52	0.53	0.51
9	3:00pm	0.54	0.54	0.54	0.56	0.56
10	4:00pm	0.52	0.54	0.53	0.51	0.54
11	5:00pm	0.51	0.53	0.51	0.52	0.50
12	6:00pm	0.53	0.54	0.54	0.53	0.53
13	7:00pm	0.55	0.56	0.53	0.54	0.56
14	8:00pm	0.56	0.55	0.53	0.50	0.54
15	9:00pm	0.54	0.51	0.52	0.51	0.50
16	10:00pm	0.55	0.56	0.55	0.54	0.55
17	11:00pm	0.52	0.52	0.53	0.53	0.52
18	12:00am	0.54	0.54	0.52	0.53	0.54
19	1:00am	0.52	0.51	0.52	0.54	0.53
20	2:00am	0.51	0.54	0.53	0.56	0.50

Table C: Showing the Data of the fill Height Measurement of Maltina in cl

Sample No.	Time	A	B	C	D	E
1	7:00am	61	59	60	61	59
2	8:00am	59	58	60	60	61
3	9:00am	60	60	60	61	58
4	10:00am	60	60	61	60	59
5	11:00am	61	59	60	58	60
6	12 noon	60	59	60	60	61
7	1:00pm	59	60	58	61	59
8	2:00pm	60	60	60	60	60
9	3:00pm	58	61	62	58	61
10	4:00pm	58	59	60	61	59
11	5:00pm	61	60	61	58	60
12	6:00pm	60	60	59	60	59
13	7:00pm	59	61	59	61	60
14	8:00pm	59	60	60	61	59
15	9:00pm	59	62	60	59	60
16	10:00pm	59	60	58	61	60
17	11:00pm	60	59	61	60	60
18	12:00am	60	61	59	60	59
19	1:00am	62	59	60	59	59
20	2:00am	59	60	60	59	62

Table D: Showing the Data of the level of co_2 in each Bottle of Maltina in wt/wt

Sample No.	Time	A	B	C	D	E
1	7:00am	0.60	0.60	0.59	0.60	0.59
2	8:00am	0.58	0.58	0.59	0.60	0.60
3	9:00am	0.59	0.60	0.60	0.61	0.59
4	10:00am	0.59	0.61	0.61	0.60	0.58
5	11:00am	0.60	0.60	0.60	0.60	0.59
6	12noon	0.59	0.59	0.59	0.60	0.60
7	1:00pm	0.60	0.60	0.61	0.60	0.60
8	2:00pm	0.60	0.61	0.60	0.59	0.59
9	3:00pm	0.59	0.61	0.60	0.59	0.60
10	4:00pm	0.60	0.60	0.60	0.60	0.61
11	5:00pm	0.62	0.61	0.61	0.60	0.60
12	6:00pm	0.61	0.61	0.61	0.60	0.61
13	7:00pm	0.61	0.62	0.60	0.61	0.60
14	8:00pm	0.62	0.60	0.60	0.61	0.62
15	9:00pm	0.60	0.61	0.60	0.61	0.60
16	10:00pm	0.61	0.59	0.60	0.61	0.60
17	11:00pm	0.60	0.60	0.60	0.60	0.61
18	12:00am	0.60	0.61	0.59	0.59	0.59
19	1:00am	0.59	0.60	0.58	0.60	0.61
20	2:00am	0.58	0.60	0.59	0.60	0.59

Table E: Showing the data of the fill height measurement of Goldberg in cl

Sample No	Time	A	B	C	D	E
1	7:00am	61	60	59	61	59
2	8:00am	59	58	60	60	61
3	9:00am	59	60	59	61	60
4	10:00am	61	60	60	59	60
5	11:00am	59	60	60	59	60
6	12 noon	61	59	60	59	61
7	1:00pm	59	60	60	58	60
8	2:00pm	60	61	59	60	60
9	3:00pm	58	61	62	58	61
10	4:00pm	58	59	60	61	59
11	5:00pm	61	60	61	58	60
12	6:00pm	60	60	59	60	59
13	7:00pm	60	59	60	61	60
14	8:00pm	57	60	61	61	60
15	9:00pm	59	61	59	60	61
16	10:00pm	59	60	58	61	60
17	11:00pm	60	59	61	60	60
18	12:00am	60	61	59	60	59
19	1:00am	59	60	59	60	61
20	2:00am	60	59	60	60	61

Table F: Showing the data of the level of co_2 in each bottle of Goldberg in wt/wt

Sample No	Time	A	B	C	D	E
1	7:00am	0.65	0.64	0.64	0.65	0.63
2	8:00am	0.63	0.63	0.64	0.65	0.63
3	9:00am	0.61	0.63	0.65	0.62	0.64
4	10:00am	0.64	0.64	0.64	0.63	0.62
5	11:00am	0.63	0.62	0.61	0.64	0.63
6	12noon	0.62	0.64	0.64	0.63	0.62
7	1:00pm	0.63	0.65	0.64	0.63	0.64
8	2:00pm	0.64	0.66	0.62	0.63	0.61
9	3:00pm	0.65	0.66	0.64	0.64	0.64
10	4:00pm	0.64	0.64	0.63	0.63	0.63
11	5:00pm	0.64	0.63	0.65	0.65	0.65
12	6:00pm	0.62	0.64	0.63	0.64	0.63
13	7:00pm	0.63	0.63	0.63	0.63	0.64
14	8:00pm	0.64	0.66	0.64	0.64	0.65
15	9:00pm	0.64	0.66	0.64	0.65	0.64
16	10:00pm	0.63	0.62	0.62	0.63	0.64
17	11:00pm	0.64	0.64	0.64	0.62	0.63
18	12:00am	0.62	0.64	0.64	0.62	0.63
19	1:00am	0.62	0.64	0.65	0.62	0.64
20	2:00am	0.62	0.63	0.63	0.63	0.64