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Quantifying Optimal Policy in an Endogenous Growth Model: A Theoretical Analysis

By Ahmed Bellakhdhar
University of Tunis

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I. Introduction

There Fiscal policy has received much attention in the literature on taxation and growth. Numerous theoretical and empirical studies have been devoted to understanding the growth and welfare effects of various taxes and government expenditures and the optimal structure of tax systems (e.g., Chamley, 1986; Barro, 1990; Turnovsky, 1996; Judd and Kenneth , 1999; Guo and Lansing, 1999; and Turnovsky, 2000). Almost all the theoretical studies in this literature use either neoclassical models or capital-based endogenous growth models. In the fully-industrialized phase three sectors are acting: the competitive final goods sector, the schooling sector where knowledge (human capital) is accumulated, and the intermediate goods sector which produces an increasing variety of goods due to R&D. In this sector there is monopolistic competition, so innovative firms charge a markup of price over cost and, therefore, production of intermediate goods is too low relative to its efficient value.

However, monopoly power is not the only plausible source of inefficiency in R&D-based growth models. Thus, empirical evidence reported, e.g., by Griliches (1992) and Porter and Stern (2000) also supports the existence of R&D spillovers in innovation −a "standing on shoulders" effect (e.g., Jones, 1995). Engelbrecht (1997) and Del Barrio-Castro, Lopez-Bazo and Serrano-Domingo (2002) find that R&D spillovers are actually statistically significant in empirical specifications that include human capital. Several authors have also pointed out that the R&D activity may be subject to an external effect associated to the duplication and overlap of research effort −a "stepping on toes" effect (e.g., Jones, 1995, Stokey, 1995). Intuitively, the larger the number of people searching for ideas is, the more likely it is that duplication of research would occur. Evidence of duplicative research has been found, e.g., by Kortum (1993) and Lambson and Phillips (2007).

According with this empirical evidence, Grossmann et al (2010), Gómez (2011) and Iacopetta (2011) have incorporated R&D spillovers in innovation and an externality associated to the duplication of research effort into the Arnold (2000a) and Funke and Strulik (2000) model. This raises the question of whether an adequate government intervention can provide the required incentives to correct these inefficiencies and make the decentralized economy to replicate the first-best solution attainable by a social planner. However, only a little number of these previous contributions has analyzed this issue. The majority of studies focus on studying the equilibrium dynamics of the market economy only. This paper seeks to fill this gap.

Author: *Higher Institute of Management, University of Tunis, Tunisia. e-mail: bellakhadar@yahoo.fr*

In Arnold (2000b) studies the optimal combination of production and R&D subsidies in the Romer (1990) model. This model has been criticised because of the implied counterfactual scale effects and, furthermore, it does not include duplication externalities. Grossmann et al. (2010b) consider instead a semi-endogenous growth model à la Jones (1995), in which economic growth is driven solely by exogenous population growth. The introduction of human capital as an additional source of growth allows to overcome this shortcoming because economic growth is fully endogenous, Gomez and T.Sequeira (2011), i.e., ultimately driven by private incentives to invest in human capital. As argued by Strulik (2007), this also reduces the importance of R&D and, therefore, the role of externalities associated to innovation. Furthermore, Grossmann et al. (2010b) do not study analytically the stability of the centrally planned economy.

Other related research has been made by Jones and Williams (2000), Alvarez-Pelaez and Groth (2005), Steger (2005) and Strulik (2007). While these works study the optimality of investments in R&D, their focus is on the quantitative assessment of distortions on the steady state −disregarding the transitional phase. Hence, the dynamic optimal policy is not analyzed. Furthermore, aside from Strulik (2007), their models do not allow for human capital accumulation. Grossmann, Steger and Trimborn (2010a) compute numerically the optimal policy in a version of the Jones (1995) model with human capital accumulation calibrated to U.S. data. However, as it is subject to diminishing returns, human capital is not a true engine of growth and it assumes a stationary long-run value. Furthermore, the optimal fiscal policy is not characterized analytically. Grossmann et al. (2010a) take into account the transition dynamics in their numerical simulations, for tractability reasons they only consider policies in which the subsidy rates are constant over time.

This paper aims to characterize analytically the optimal dynamic fiscal policy in R&D-based endogenous growth model which incorporates domestic innovation, investment in education, distance to technology frontier and external technology spillovers through import of technologically advanced products and foreign direct investment as engines of growth. The model incorporates three sources of inefficiency: monopolistic competition in the intermediate-goods sector, duplication externalities and spillovers in R&D. To this end, we analyze the efficient growth path that a benevolent social planner would implement. We aim to provide conditions for the existence of a unique feasible optimal steady state with positive long-run growth. The optimal growth path can be decentralized by means of a tax on capital income at a constant rate combined with equality between the share of public spending in the total expenditure on education net of subsidy and the tax on labor income and a time-varying subsidy to R&D which addresses the duplication externalities and spillovers in R&D associated to the innovation process. Unlike previous works that rely solely on steady-state analysis, we take explicitly into account the transitional dynamics when evaluating the economic effect of removing the inefficiencies.

The remainder of this paper is organized as follows. Section 2 describes the decentralized economy. Section 3 analyzes the socially planned economy. Section 3 devises an optimal fiscal policy capable of decentralizing the optimal growth path and Section 4 concludes.

II. The Market Economy

Consider an economy where total supply of labour is constant ($L_t = L, \forall t$). It consists of an education sector knowledge (human capital) is accumulated and three other productive sectors: a final goods sector, an intermediate goods sector, and finally, a research sector. While the final goods sector and the R&D sector are competitive, the intermediate goods sector is monopolistic. The endowment of time is normalized as a constant flow of one unit per period. A fraction u_y of time is devoted to production of final goods, a fraction u_h to education, and a fraction $u_R = 1 - u_y - u_h$ to innovation activities.

The market for final goods is perfectly competitive and the price for final goods is normalized to one. Final output, Y is produced with a Cobb-Douglas technology

$$Y = \left(u_y H\right)^{1-\alpha} \int_{i=O}^{A} x_i^{\alpha} di\,, \qquad 0 < \alpha < 1 \tag{1}$$

Where, H is the level of total human capital, $(1-\alpha)$ is the human capital's income share and x_{it} is the amount used for each one of the A intermediategoods. To enter the intermediate sector, a firm must acquire a patent from the successful innovator which allows the firm to produce an improved differentiated intermediate by employing physical capital K and charge a monopoly price for the product. In the sector i, the production function of the quantity x_{it} is specified as $x_i = K/A$. Profit maximization delivers the factor demands as follow: The interest rate $(r = \alpha^2 Y/K)$, the wage rate per unit of employed human capital $\left(w = (1-\alpha)Y/u_y H\right)$ and the price of the i^{th} intermediate goods $\left(p_i = \alpha Y x_i^{\alpha-1} / \int_{i=O}^{A} x_i^{\alpha} di\right)$.

Each firm in the intermediate goods sector owns an infinitely-lived patent for selling its variety x_i, which costs r unit of Y to be produced. For each unit sold of the intermediate goods producers receive a unit price p_i. Producers act under monopolistic competition and maximize operating profits: $\pi_i = (p_i - r)x_i$. Profit maximization in this sector implies that each firm charges a price of $(p_i = r/\alpha)$. Under symmetric hypothesis, we have $x_i = x$ and $p_i = p\ \forall i$. Hence, the quantity of intermediates employed is $xA = \alpha^2 Y/r$, firm profit is $\pi_i = (1-\alpha)\alpha\, Y/A$ and $\int_{i=O}^{A} x_i^{\alpha} di = Ax^{\alpha}$. Substituting this expression into (1) yields $y = k^{\alpha}\left(Au_y h\right)^{1-\alpha}$. Where, y, k and h are the final output, physical capital and human capital per worker, respectively.

A representative household derives utility from consumption, c according to

$$\int_{0}^{\infty} \frac{c_t^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt, \quad \rho > 0 \tag{2}$$

Where, ρ is the rate of time preference and σ is the relative risk aversion. His human capital is accumulated according to:

$$\dot{h}_h = B(u_h h_h)^{\vartheta} \overline{D}_h^{\,1-\vartheta} \tag{3}$$

Here, B is a positive technical parameter determining at what rate investments in the education sector are converted to a growth human capital, $\overline{D}$ is the private expenditure on education per student and $(0 < \vartheta < 1)$ captures decreasing returns to teaching input. The fraction u_h is not directly observed. It' modeled in many studies by the ratio of the average number of years of schooling S to the life expectancy L_e; $u_h \approx (S/L_e)$. The budget constraint faced by a representative individual is given by the following equation:

$$\dot{a} = (1-\tau_k)ra + (1-\tau_w)w(1-u_h)h_h - c - (1-s_d)\overline{D} \tag{4}$$

Where, a is the average wealth, τ_k, τ_w and s_d are taxes on capital and labor incomes and education subsidy accorded by the government. Empirical evidence shows that both types of school expenditure (private and public) are proportional on average. We then assume a linear relationship between the two variables defined as follows: $D_{priv} \approx \ell D_{pub}$, where ℓ is a positive constant.

Let g_x denote x's growth rate, $g_x = \dot{x}/x$ and x_0 the initial value of the variable x. The individual maximizes her intertemporal utility (1), subject to the human capital accumulation technology (3) and the budget constraint (4). The resolution of this program gives:

$$\begin{cases} logh = logh_0 + B\left(\ell \times \frac{u_y}{u_h}\right)^{1-\vartheta}\left(\frac{y_0}{h_0}\right)^{1-\vartheta}\left(\frac{D_{pub}}{Y}\right)^{1-\vartheta} u_h & (5) \\ g_h = \frac{dlog(h)}{dt} \approx \frac{log(h_t) - log(h_0)}{\Delta t} = \underbrace{\vartheta B\left(\ell \times \frac{u_y}{u_h}\right)^{1-\vartheta}\left(\frac{y_0}{h_0}\right)^{1-\vartheta}}_{\alpha_h}\left(\frac{D_{pub}}{Y}\right)^{1-\vartheta} u_h & (6) \end{cases}$$

This result shows that the education subsidy stimulates human capital accumulation, whereas the tax on labor income has a negative impact. This confirms the empirical evidence provided by Hanushek and Kimko (2000) and Pritchett (2001), Marcelo Soto (2006) and Florent (2016)

From these equations, we deduce that the aggregate human capital H acquired through education can be expressed as follow:

$$H = H_0 \times e^{\alpha_h \left(\frac{D_{pub}}{Y}\right)^{1-\vartheta} S} \tag{7}$$

Where, $\left(\frac{D_{pub}}{Y}\right)$ is the total public expenditure on education expressed as a percentage of GDP (*Index of Education Quality*) and α_h is the rate of return to schooling corrected by the quality index.

In the R&D sector, the invention of new intermediates is determined according to

$$\dot{A} = \delta' \underbrace{(u_R h)^{\theta}}_{\text{Domestic innovation}} \underbrace{\left(\frac{M}{Y}\right)^{\epsilon}\left(\frac{FDI}{Y}\right)^{\tau}}_{\text{Technology spillovers}} \underbrace{\left(\frac{A_{sup} - A}{A_{sup}}\right)^{\gamma}}_{\text{Distance to frontier}} \underbrace{A^{\emptyset}}_{\text{Externality effect}} \tag{8}$$

Where, $\delta' > 0$ is a parameter of research productivity and $(u_R h)$ represents average human capital devoted to innovation. Hence, this specification incorporates a duplication externality of research effort, as well as the potential for spillovers in R&D. We assume that $0 \leq \theta < 1$ and $0 \leq \emptyset < 1$. The fraction u_R is approximated by the proportion of scientists and engineers engaged in R&D L to the total labor force L (see Ha and Howitt,2007; Madsen, 2008; Madsen et al., 2010). It is parameterized by the variable$\left(\frac{L_R}{L} \approx u_R\right)$. A_{sup} is frontier technology, and measures the available "leading-edge technology" and $\left(\frac{A_{sup}-A}{A_{sup}}\right)$ is the relative difference in total factor productivity of an economy from the global maximum. This term captures the idea that there are benefits to backwardness. M is nominal import of technologically advanced products from the industrial countries and (FDI/Y) is the share of inward FDI flows in GDP. In this model, we divide by GDP to allow for product proliferation and increasing complexity of new innovations as productivity increases (Ha and Howitt, 2007).

Since developing countries carry out little or, insignificant R&D activities, the degree of technological diffusion from countries close to the frontier is likely to be one of the key drivers to accelerate the TFP growth in those developing economies (Savvides and Zachariadis, 2005). Coe *et al*. (1997) argue that total factor productivity in developing countries is positively and significantly related to R&D in their industrial country trade partners and to their import of technology. Innovation is usually embodied in capital and intermediate goods and therefore the direct import of these goods is one channel of international technology spillovers (Grossman and Helpman, 1991; Coe and Helpman, 1995). Foreign Direct Investment (FDI) by the Multinational Corporations (MNCs) may be another channel for the international transmission of technology (Savvides and Zachariadis, 2005).

The rate of the subsidy to R&D is noted by s_R. This means that $(1-s_R)$ represents the proportion of costs that are supported by the firm. Innovative firm profit is

$$\pi = \dot{A}V - \underbrace{[(1-s_R)R + \alpha_m . M]}_{\hat{C}_{Tinv}} \tag{9}$$

Where, $R = wH_R = wL_R h$, V is the value of an innovation and $\hat{C}_{Tinv}$ is the total cost supported by the firm. α_m is a positive constant inferior to the unity. An innovation is worth the present value of the stream of monopoly profits $V_t = \int_t^{\infty} e^{\int_t^{\tau} r(s)ds}\pi(\tau)d\tau$. Differentiating this expression with respect to time yields the no-arbitrage equation $g_v = r - \pi/V$.

The government may subsidize education and R&D costs and accord fiscal advantages to Multinational Firms to attract foreign investment, financed by the sum of taxes on labor and physical capital incomes, so that its budget constraint is

$$\tau_k raL + \tau_w w(1-u_h)H = \alpha_d FDI + s_d D_{priv} + D_{pub} + s_R w u_R H \tag{10}$$

In this equation, the left side is the state's fiscal resources. These are taxes collected on wages $(\tau_w w(1-u_h)H)$ and on capital income $(\tau_k raL)$.The right-hand side represents the expenses supported by the state in the form of tax incentives or financial charges for the attraction of foreign direct investment $(\alpha_d FDI)$, public expenditure on education (D_{pub}) and the subsidy of total private school expenditure $(s_d D_{priv})$ and a subsidy of the total R&D cost $(s_R w u_R H)$. This constraint is assumed balanced at each period. Here, the principal of the state is to determine the optimal Mix (subsidies and taxes) that maximize social welfare.

Let $\chi \equiv \frac{C}{K}$ denote the consumption to physical capital ratio, and $\psi \equiv h^{\theta} A^{\emptyset - 1}$, the knowledge-ideas ratio. Physical capital and claims to innovative firms are the assets in the economy. Aggregate wealth is then $aL = K + AV$. The equilibrium dynamics of the market economy in terms of the variables r, χ, u_y, ψ and g_A is determined by:

$$g_r^* = \left(\frac{1-\alpha}{\alpha}\right)\left(\vartheta^2 B(1-\alpha)^{1-\vartheta}\left(\frac{1-\vartheta}{\vartheta}\right)^{1-\vartheta}\left(\frac{1-\tau_w}{1-s_d}\right)^{1-\vartheta}\left(\frac{y_0}{h_0}\right)^{1-\vartheta} - (1-\tau_k)r\right) + \left(\frac{1-\alpha}{\alpha}\right) g_A \tag{11}$$

$$g_\chi^* = \frac{r^*}{\alpha^2}\left[\frac{\alpha^2(1-\tau_k)}{\sigma} + (1-\alpha)\left(\frac{1+\ell}{\ell}\right)\left(\frac{1-\vartheta}{\vartheta}\right)\left(\frac{1-\tau_w}{1-s_d}\right)\frac{u_h^*}{u_y^*} + \frac{R_d}{Y} + \frac{R_m}{Y} - 1\right] - \frac{\rho}{\sigma} + \chi \tag{12}$$

$$g_{u_y}^* = \frac{r}{\alpha^2}\left[1 - \frac{R_d}{Y} - \frac{R_m}{Y} - \alpha^2(1-\tau_k) - (1-\alpha)\left(\frac{1+\ell}{\ell}\right)\left(\frac{1-\vartheta}{\vartheta}\right)\left(\frac{1-\tau_w}{1-s_d}\right)\frac{u_h}{u_y}\right] - \chi - \left(1 - \frac{\vartheta}{u_h}\right) g_h \tag{13}$$

$$g_{\psi}^{*} = \theta\vartheta B(1-\alpha)^{1-\vartheta}\left(\frac{1-\vartheta}{\vartheta}\right)^{1-\vartheta}\left(\frac{1-\tau_w}{1-s_d}\right)^{1-\vartheta}\left(\frac{y_0}{h_0}\right)^{1-\vartheta} u_h - (1-\emptyset)g_A \tag{14}$$

$$g_{g_A}^{*} = g_h\left(1-\frac{\vartheta}{u_h}\right) + g_A\left[\left(\frac{\theta\alpha}{1-s_R}\right)\frac{u_y}{u_R} - 1\right] - \tau_k r - \frac{\dot{s}_R}{1-s_R} \tag{15}$$

If $(s_R = 0)$, so that $(\dot{s}_R = 0)$, we obtain the system that describes the dynamics of the market economy in the absence of government intervention analyzed by Gómez (2011). Proceeding in a similar manner as there, taking into account that the optimal subsidies have to be constant in the long-run $(\dot{s}_R = 0)$, the steady state of the market economy is given by:

$$r^{*} = \frac{\sigma(\mho+1)\vartheta^2 B(1-\alpha)^{1-\vartheta}\left(\frac{1-\vartheta}{\vartheta}\right)^{1-\vartheta}\left(\frac{y_0}{h_0}\right)^{1-\vartheta}\left(\frac{1-\tau_w}{1-s_d}\right)^{1-\vartheta} - \rho}{(1-\tau_k)[\sigma(\mho+1)-1]} \tag{16}$$

$$\chi^{*} = \frac{\rho}{\sigma} - \frac{r^{*}}{\alpha^2}\left[\frac{\alpha^2(1-\tau_k)}{\sigma} + (1-\alpha)\left(\frac{1+\ell}{\ell}\right)\frac{1-\vartheta}{\vartheta}\frac{(1-\tau_w)}{(1-s_d)}\frac{u_h^{*}}{u_y^{*}} + \frac{R_d}{Y} + \frac{R_m}{Y} - 1\right] \tag{17}$$

$$g_A^{*} = \frac{\vartheta}{\sigma(\mho+1)-1}\left[\vartheta B(1-\alpha)^{1-\vartheta}\left(\frac{1-\vartheta}{\vartheta}\right)^{1-\vartheta}\left(\frac{y_0}{h_0}\right)^{1-\vartheta}\left(\frac{1-\tau_w}{1-s_d}\right)^{1-\vartheta} - \frac{\rho}{\vartheta}\right] \tag{18}$$

$$g_h^{*} = \mho g_A^{*} \tag{19}$$

$$u_R^{*} = \frac{1 - \frac{\vartheta\mho}{\sigma(\mho+1)-1}\left[1 - \frac{\rho}{\vartheta^2 B(1-\alpha)^{1-\vartheta}\left(\frac{1-\vartheta}{\vartheta}\right)^{1-\vartheta}\left(\frac{1-\tau_w}{1-s_d}\right)^{1-\vartheta}\left(\frac{y_0}{h_0}\right)^{1-\vartheta}}\right]}{1 + \frac{(1-s_R)}{\theta\alpha}\left\{\left[\frac{\sigma(\mho+1)}{1-\tau_k} + \frac{\rho[\sigma(\mho+1)-1]/(1-\tau_k)}{\vartheta^2 B(1-\alpha)^{1-\vartheta}\left(\frac{1-\vartheta}{\vartheta}\right)^{1-\vartheta}\left(\frac{1-\tau_w}{1-s_d}\right)^{1-\vartheta}\left(\frac{y_0}{h_0}\right)^{1-\vartheta} - \rho}\right] - \mho\right\}} \tag{20}$$

$$u_y^{*} = \frac{(1-s_R)u_R^{*}}{\theta\alpha}\left(\frac{r^{*}}{g_A^{*}} - \mho\right) \tag{21}$$

$$\psi^{*} = g_A^{*}\Big/\left[\delta\, u_R^{\theta}\left(\frac{M}{Y}\right)^{\epsilon}\left(\frac{FDI}{Y}\right)^{\tau}\left(\frac{A_{sup}-A}{A_{sup}}\right)^{\gamma}\right] \tag{22}$$

$$g_y^{*} = g_c^{*} = g_k^{*} = \left[1+\frac{1}{\mho}\right]\vartheta B(1-\alpha)^{1-\vartheta}\left(\frac{1-\vartheta}{\vartheta}\right)^{1-\vartheta}\left(\frac{y_0}{h_0}\right)^{1-\vartheta}\left(\frac{1-\tau_w}{1-s_d}\right)^{1-\vartheta} u_h^{*} \tag{23}$$

Where, $\mho = \left(\frac{1-\emptyset}{\theta}\right)$. In this model, long-run growth depends on fiscal policy parameters.

III. The Socially Planned Economy

The social planner possesses complete information and chooses all quantities directly, taking all the relevant information into account. Since the intermediate-goods sector is symmetric, the production function can be rewritten as $Y = K^{\alpha}(Au_yH)^{1-\alpha}$, and the economy's resources constraint is $\dot{K} = K^{\alpha}(Au_yH)^{1-\alpha} - C - (1+\ell)D_{pub} - \alpha_m M - \alpha_d FDI$, given that $D_{Totale} = (1+\ell)D_{pub}$. The human capital accumulation can be rewritten in the aggregate form as follow: $\dot{H} = B[(1-u_y-u_R)H]^{\vartheta}(\ell D_{pub})^{1-\vartheta}$.

The social planner seeks to maximize (2) in aggregate form subject to the resources' constraint $(\dot{K} > 0)$, knowledge formation $(\dot{H} > 0)$ and technologies $(\dot{A} > 0)$. Let $\mathcal{H}$ be the current value Hamiltonian of the planner's maximization problem, and let λ, $\aleph$ and μ be the multipliers for the three constraints, respectively:

$$\begin{aligned}\boldsymbol{\mathcal{H}} = \frac{C_t^{1-\sigma}-1}{1-\sigma} &+ \boldsymbol{\lambda_t}[u_y^{1-\alpha}H_t^{1-\alpha}A_t^{1-\alpha}K_t^{\alpha} - C_t - (1+\ell)D_{pub,t} - \alpha_m M_{jt} - \alpha_d FDI_t] \\ &+ \boldsymbol{\aleph_t}\left[\delta\varepsilon^{\theta}\, u_R^{\theta}\left(\frac{1}{L_t}\right)^{\theta}\left(\frac{M_{jt}}{Y_t}\right)^{\epsilon}\left(\frac{FDI_t}{Y_t}\right)^{\tau}\left(\frac{A_{sup}-A_t}{A_{sup}}\right)^{\gamma} H_t^{\theta}A_t^{\phi}\right] \\ &+ \boldsymbol{\mu_t}\left[B[(1-u_y-u_R)H_t]^{\vartheta}(\ell D_{pub,t})^{1-\vartheta}\right]\end{aligned}$$

Here, the control variables are C, D, u_y , u_R, M and FDI, and the state variables, K, H and A. We focus on a fully industrialized economy characterized by the presence of physical capital accumulation, human capital formation and R&D.

The first order conditions for an interior solution

$$\frac{d\boldsymbol{\mathcal{H}}}{dC} = 0 \qquad \Rightarrow C_t^{-\sigma} = \lambda_t \tag{a}$$

$$\frac{d\boldsymbol{\mathcal{H}}}{dD_{pub,t}} = 0 \qquad \Rightarrow \mu_t(1-\vartheta)\frac{H_t}{D_{pub,t}}g_{H_t} = (1+\ell)\lambda_t \tag{b}$$

$$\frac{d\boldsymbol{\mathcal{H}}}{du_y} = 0 \qquad \Rightarrow \mu_t\vartheta\frac{H_t}{(1-u_y-u_R)}g_{H_t} = \lambda_t(1-\alpha)\frac{Y_t}{u_y} \tag{c}$$

$$\frac{d\boldsymbol{\mathcal{H}}}{du_R} = 0 \qquad \Rightarrow \mu_t\vartheta\frac{H_t}{(1-u_y-u_R)}g_{H_t} = \aleph_t\theta\frac{A_t}{u_R}g_{A_t} \tag{d}$$

$$\frac{d\boldsymbol{\mathcal{H}}}{dM_j} = 0 \qquad \Rightarrow \alpha_m M_{jt} = \frac{\aleph_t}{\lambda_t}\epsilon A_t g_{A_t} \tag{e}$$

$$\frac{d\boldsymbol{\mathcal{H}}}{dIDE} = 0 \qquad \Rightarrow \alpha_d IDE_t = \frac{\aleph_t}{\lambda_t}\tau A_t g_{A_t} \tag{f}$$

Resources' Constraints

$$\frac{d\boldsymbol{\mathcal{H}}}{dK} = \rho\lambda_t - \dot{\lambda}_t \Rightarrow \frac{\dot{\lambda}_t}{\lambda_t} = \rho - \alpha\frac{Y_t}{K_t} \tag{g}$$

$$\frac{d\boldsymbol{\mathcal{H}}}{dH} = \rho\mu_t - \dot{\mu}_t \Rightarrow \frac{\dot{\mu}_t}{\mu_t} = \rho - \frac{1}{\mu_t}\frac{d\boldsymbol{\mathcal{H}}}{dH} \tag{h}$$

$$\frac{d\mathcal{H}}{dA} = \rho\aleph_t - \dot{\aleph}_t \quad \Rightarrow \frac{\dot{\aleph}_t}{\aleph_t} = \rho - \frac{1}{\aleph_t}\frac{d\mathcal{H}}{dA} \qquad (i)$$

Transversality Conditions

$$\lim_{t\to\infty} e^{-\rho t}\lambda_t K_t = 0,\ \lim_{t\to\infty} e^{-\rho t}\mu_t H_t = 0,\ \lim_{t\to\infty} e^{-\rho t}\aleph_t A_t = 0 \qquad (j)$$

There are two main qualitative differences between the equilibrium outcome of a decentralized economy and the first-best optimum attainable by a social planner. First, the social planner internalizes the inefficiency due to the presence of monopolistic competition in intermediate-goods production. Therefore, he chooses to devote to intermediate-goods production a fraction of output equal to the square of the elasticity of intermediates in the production of the final good multiplied by the interest rate, $xA/Y = \alpha^2 r$. Second, the social planner internalizes the spillovers in R&D and the duplication externalities that are present in the innovation process. Thus, this is taken into account when choosing the optimal fraction of time devoted to innovation and when setting the optimal shadow value of an innovation.

In balanced growth path (or steady state) all variables grow at constant but possibly different rates, and the shares of labor in its different uses are constant. We can state the following proposition. We associate the index (^) to indicate social equilibrium's solutions.

Proposition 1. *Let* $\vartheta^2 B(1-\alpha)^{1-\vartheta}\left(\frac{1-\vartheta}{\vartheta}\right)^{1-\vartheta}\left(\frac{\ell}{1+\ell}\right)^{1-\vartheta}\left(\frac{y_0}{h_0}\right)^{1-\vartheta} > \rho$. *The socially planned economy has a unique positive steady state with positive long-run growth, in which the interest rate is*

(i) $$\hat{r} = \alpha\left[\frac{\sigma(\mho+1)\vartheta^2 B(1-\alpha)^{1-\vartheta}\left(\frac{1-\vartheta}{\vartheta}\right)^{1-\vartheta}\left(\frac{\ell}{1+\ell}\right)^{1-\vartheta}\left(\frac{y_0}{h_0}\right)^{1-\vartheta} - \rho}{[\sigma(\mho+1)-1]}\right]$$

A positive long-run growth rates of GDP, de consumption and physical capital

(ii) $$\hat{g}_C = \hat{g}_K = \hat{g}_Y = \frac{(\hat{r}/\alpha)-\rho}{\sigma} = \left[\frac{1+\mho}{\sigma(1+\mho)-1}\right]\left[\vartheta^2 B(1-\alpha)^{1-\vartheta}\left(\frac{1-\vartheta}{\vartheta}\right)^{1-\vartheta}\left(\frac{\ell}{1+\ell}\right)^{1-\vartheta}\left(\frac{y_0}{h_0}\right)^{1-\vartheta} - \rho\right]$$

If and only if $\sigma > \sigma_{min} = \frac{1}{(1+\mho)}$

Long-run growth rate of technology

(iii) $$\hat{g}_A = \left(\frac{1}{\mho+1}\right)\hat{g}_y = \frac{1}{\sigma(1+\mho)-1}\left[\vartheta^2 B(1-\alpha)^{1-\vartheta}\left(\frac{1-\vartheta}{\vartheta}\right)^{1-\vartheta}\left(\frac{\ell}{1+\ell}\right)^{1-\vartheta}\left(\frac{y_0}{h_0}\right)^{1-\vartheta} - \rho\right]$$

Long-run growth rate of human capital

(iv) $$\hat{g}_h = \mho\hat{g}_A = \frac{\mho}{\sigma(1+\mho)-1}\left[\vartheta^2 B(1-\alpha)^{1-\vartheta}\left(\frac{1-\vartheta}{\vartheta}\right)^{1-\vartheta}\left(\frac{\ell}{1+\ell}\right)^{1-\vartheta}\left(\frac{y_0}{h_0}\right)^{1-\vartheta} - \rho\right]$$

Investment rate in physical capital

(v) $$\widehat{Inv}_K = \frac{\alpha}{\sigma}\left[1 - \frac{[\sigma(1+\mho)-1]\rho}{\sigma(1+\mho)\vartheta^2 B(1-\alpha)^{1-\vartheta}\left(\frac{1-\vartheta}{\vartheta}\right)^{1-\vartheta}\left(\frac{\ell}{1+\ell}\right)^{1-\vartheta}\left(\frac{y_0}{h_0}\right)^{1-\vartheta} - \rho}\right]$$

The consumption to physical capital ratio

(vi) $$\hat{\chi} = \frac{\rho}{\sigma} + \frac{\vartheta^2 B\rho - \sigma(1+\mho)(1-\alpha)^{1-\vartheta}\left(\frac{1-\vartheta}{\vartheta}\times\frac{\ell}{1+\ell}\times\frac{y_0}{h_0}\right)^{1-\vartheta}}{\alpha[\sigma(\mho+1)-1]}\left[\frac{\alpha}{\sigma} + \frac{(1-\alpha)(1-\vartheta)\left(\frac{\hat{u}_h}{\hat{u}_y}\right)}{\vartheta} + \frac{R_d}{Y} + \frac{R_m}{Y} - 1\right]$$

Fractions of time devoted to education, R&D and final production, respectively

(vii) $$\hat{u}_h = \frac{\vartheta\mho}{\sigma(\mho+1)-1}\left[1 - \frac{\rho}{\vartheta^2 B(1-\alpha)^{1-\vartheta}\left(\frac{1-\vartheta}{\vartheta}\right)^{1-\vartheta}\left(\frac{\ell}{1+\ell}\right)^{1-\vartheta}\left(\frac{y_0}{h_0}\right)^{1-\vartheta}}\right]$$

And

(viii) $$\hat{u}_R = \frac{1-\frac{\vartheta\mho}{\sigma(\mho+1)-1}\left[1-\frac{\rho}{\vartheta^2 B(1-\alpha)^{1-\vartheta}\left(\frac{1-\vartheta}{\vartheta}\right)^{1-\vartheta}\left(\frac{\ell}{1+\ell}\right)^{1-\vartheta}\left(\frac{y_0}{h_0}\right)^{1-\vartheta}}\right]}{1+\frac{1}{\theta}\left[\sigma(\mho+1)-\mho+\frac{\rho[\sigma(\mho+1)-1]}{\vartheta^2 B(1-\alpha)^{1-\vartheta}\left(\frac{1-\vartheta}{\vartheta}\right)^{1-\vartheta}\left(\frac{\ell}{1+\ell}\right)^{1-\vartheta}\left(\frac{y_0}{h_0}\right)^{1-\vartheta}-\rho}+\gamma\left(\frac{A_t}{A_{max}-A_t}\right)-\emptyset\right]}$$

(ix) $$\hat{u}_y = 1 - \hat{u}_h - \hat{u}_R$$

Comparing the optimal steady-state values in Proposition 1 with their corresponding equilibrium values in the market economy given by (16) - (23) in the absence of government intervention, $s_R = s_d = \tau_w = \tau_k = 0$, we observe that the long-run equilibrium growth rates of consumption, output, physical capital, human capital and the number of product varieties, as well as the time devoted to education, in the market economy coincide with their stationary optimal values. Long-run distortions only arise in the ratio of consumption to physical capital, χ, the interest rate, and the fractions of time devoted to production and innovation, u_y and u .

The steady-state ratio of consumption to physical capital is too high in the market equilibrium, reflecting the fact that the production of intermediate goods is too low due to monopolistic competition in this sector. However, the relationship between the long-run equilibrium and optimal shares of labor devoted to production and innovation is ambiguous. R&D spillovers cause the equilibrium share of labor devoted to innovation to be too low relative to its optimum value. The suboptimal low production of intermediates due to markup pricing has a similar effect. However, duplication externalities have the opposite effect and would make the market economy to overinvest in R&D. Thus, the overall effect depends on the relative values of the externalities associated to the R&D process, as well as on the size of the markup.

IV. Market Inefficiencies and Optimal Policies: Theoretical Analyzes

Theoretical analyzes show the existence of some market distortions. The first one is linked to the presence of imperfect competition in the intermediate goods sector. The second inefficiency results from the knowledge externality that affects technology. While innovation is a source of social surplus in the R&D sector, this surplus is not entirely appropriate by innovators. However, the existence of non-internalized externalities by the decision-makers can lead to non optimal solutions. To correct these imperfections, the intervention of the state by an effective fiscal policy is necessary. More specifically, the state must choose the appropriate policy variables that allow the decentralized economy to achieve sustainable optimal growth. To better understand this phenomenon, several theoretical analyzes need to be developed.

a) Physical capital investment

At equilibrium, the demand function of the intermediate good is defined by:

$$x_i^* = \left(\frac{\alpha^2}{r}\right)^{1/_{1-\alpha}} u_y H$$

This latter relationship shows that a high real interest rate discouraged the demand for intermediate goods by the producer of the final good. In other hand, a strong monopolistic competition (α is low), the cost of using intermediate goods in final production $\left(p_i = \frac{r}{\alpha}\right)$ is so higher. This can lead to a decrease in their demand. In the long run, this phenomenon can lead to a reduced investment rate (underinvestment in K), which in turn leads to a decrease in final output. However, monopolistic competition can have negative effects on the accumulation of physical capital and, in turn, on economic growth.

To correct this negative effect, the state can act through several effective policies. Any policy that reduces the cost of using physical capital or motivates households to save more will be beneficial for growth. Empirical studies show that the attraction of FDI, economic openness, an important subsidy of school expenses and a reduced tax on incomes are some of the most favorable policies. Our main objective here is to understand the role that the state can play in dealing with monopoly distortions through optimal tax policy. At market equilibrium, the real interest rate is defined by:

$$r^* = \frac{1}{(1-\tau_k)} \times \left[\frac{\sigma(\mho+1)\vartheta^2 B(1-\alpha)^{1-\vartheta}\left(\frac{1-\vartheta}{\vartheta}\right)^{1-\vartheta}\left(\frac{1-\tau_w}{1-s_d}\right)^{1-\vartheta}\left(\frac{y_0}{h_0}\right)^{1-\vartheta} - \rho}{\sigma(\mho+1)-1}\right]$$

This expression shows that the two tax variables τ_w and τ_k have opposite impacts on the real interest rate. An increase in τ_k creates an augmentation in the cost of the physical capital, whereas the taxation of wages has opposite effects. This theoretical result was explained by Judd (1987).

We denote by x^{LF}, the optimal solutions of the laissez-faire equilibrium. They are exactly the solutions found at market equilibrium but with zero fiscal variables. Based on this definition, our analytical results show that the ratio $\left(\frac{\hat{r}}{r^{LF}}\right)$ is found less than unity. However, without the intervention of the state through an effective policy, the real interest rate remains very higher than its optimal value.

At the decentralized equilibrium, if we replace $r^*$by its expression in the investment rate defined by $Inv = \frac{\dot{K}}{Y}$, we obtain the following expression:

$$Inv^* = \frac{\alpha^2}{\sigma}(1-\tau_k)\left[1 - \frac{[\sigma(\mho+1)-1]\rho}{\sigma(\mho+1)\vartheta^2 B(1-\alpha)^{1-\vartheta}\left(\frac{1-\vartheta}{\vartheta}\right)^{1-\vartheta}\left(\frac{1-\tau_w}{1-s_d}\right)^{1-\vartheta}\left(\frac{y_0}{h_0}\right)^{1-\vartheta} - \rho}\right]$$

This expression shows that the subsidy of education can have an indirect positive effect on the rate of investment in physical capital but all types of taxation have a negative impact. In other words, education subsidy motivates households to save more but high taxes discourage physical capital accumulation. Companies will therefore have limited access to new technologies that require less labor. As a result, labor productivity will fall, which reduces the growth rate of output per worker.

For zero tax variables, the investment rate in physical capital is expressed as:

$$Inv^{LF} = \frac{\alpha^2}{\sigma}\left[1 - \frac{[\sigma(\mho + 1) - 1]\rho}{\sigma(\mho + 1)\vartheta^2 B(1-\alpha)^{1-\vartheta}\left(\frac{1-\vartheta}{\vartheta}\right)^{1-\vartheta}\left(\frac{y_0}{h_0}\right)^{1-\vartheta} - \rho}\right]$$

Since $0 < \alpha < 1$, and $\frac{\ell}{1+\ell} < 1$, then the comparison between the optimal rate of investment in physical capital and its level with zero tax remains ambiguous. The optimal rate of investment is obtained for $\left(\frac{1-\bar{\tau}_w}{1-\bar{s}_d}\right) = \left(\frac{D_{priv}}{D_{Totale}}\right) \approx \left(\frac{\ell}{1+\ell}\right)$ and $\left(\tau_k = 1 - \frac{1}{\alpha}\right)$. It is the optimal Tax-Mix to achieve optimal level of this type of capital.

Our theoretical results also show that the subsidy of education s_d can improve the rate of investment in physical capital in an indirect way through the reduction of school expenses supported by households. Thus, the state can react through this type of subsidy to correct imperfections of underinvestment in physical capital and technology. This idea is also identified in the following aggregate constraint:

$$\dot{a}L = \underbrace{\dot{K}}_{\text{Physical capital accumulation}} + \underbrace{\left(\dot{A}V + A\dot{V}\right)}_{\text{Investment in technology}}$$

These results constitute to my knowledge a contribution in the literature of endogenous growth.

b) Human capital investment

At the decentralized equilibrium, the fraction of time devoted to education is expressed by:

$$u_h^* = \frac{\vartheta\mho}{\sigma(\mho + 1) - 1}\left[1 - \frac{\rho}{\vartheta^2 B(1-\alpha)^{1-\vartheta}\left(\frac{1-\vartheta}{\vartheta}\right)^{1-\vartheta}\left(\frac{1-\tau_w}{1-s_d}\right)^{1-\vartheta}\left(\frac{y_0}{h_0}\right)^{1-\vartheta}}\right]$$

This equation shows that an increase in the tax rate τ_w has negative effect on the investment in education (under-investment in human capital), while education subsidy encourages households to devote more time to education.

At the market equilibrium, the growth rate of human capital is expressed as follows:

$$g_h^* = \vartheta B(1-\alpha)^{1-\vartheta}\left(\frac{1-\vartheta}{\vartheta}\right)^{1-\vartheta}\left(\frac{1-\tau_w}{1-s_d}\right)^{1-\vartheta}\left(\frac{y_0}{h_0}\right)^{1-\vartheta} u_h^* = \left(\frac{\mho}{\mho + 1}\right) g_y^*$$

From this equation, we remark that taxation of wages has a negative impact on the accumulation of skills and, in turn, on economic growth. These negative repercussions can be corrected by a high education subsidy. The optimal growth rate of human capital is achieved for equality between the ratio $\left(\frac{1-\tau_w}{1-s_d}\right)$ and the share of private expenditure in total expenditure on education. In other words, the negative impact caused by the taxation of wages must be offset by the education subsidy.

The analytical development of the expression of g_h^* shows that the growth rate of human capital can be expressed as a function of the investment rate as follows:

$$g_h^* = \frac{1}{\vartheta\sigma(\mho+1)}\left[\frac{[\sigma(\mho+1)-1]\rho}{1-\frac{\sigma}{\alpha^2}\frac{Inv^*}{(1-\tau_k)}}+\rho\right]u_h^*$$

This new expression shows that the rate of growth of human capital depends positively on the rate of investment in physical capital. A high investment rate is a favorable condition for skill accumulation. This theoretical result confirms the empirical evidence found by Judson (2002) that in rich countries, the level of human capital is relatively higher than in poor countries. This proves the strong complementarity between the two types of capitals.

To understand the imperfections related to monopolistic competition and the role that the state can play by its own policies to stimulate investment in R&D, we will take as a starting point the non-arbitrage condition in the R&D sector.

Let π_A the profit research firm. It is defined by the following equation:

$$\pi_A = \dot{A}\int_0^t \pi_{ix}dx - (1-s_R)R - \alpha_m.M$$

Although innovation is a source of social surplus, innovators may not internalize this positive externality in their decisions. This distortion linked to the externality of knowledge can affect the production of technology and lead to suboptimal solutions.

The economic surplus resulting from R&D is defined theoretically by $\left(\frac{dY_t}{dA_t} = (1-\alpha)\frac{Y_t}{A_t}\right)$, while the profit of a monopoly is expressed by $\pi_t^* = \alpha(1-\alpha)\frac{Y_t}{A_t} < (1-\alpha)\frac{Y_t}{A_t} \equiv Real\ Economic\ Surplus$. This inequation shows that for a very small α (strong monopolistic competition), innovative firms only consider a small part of the economic surplus. As a result, the existence of non-internalized externalities can lead to the prediction of a reduced present value of profits of intermediate goods V_t and, in turn, to an underinvestment in technology.

c) R&D investment

At market equilibrium, the optimal fraction of the time devoted to R & D is expressed by:

$$u_R^* = \frac{1-\frac{\vartheta\mho}{\sigma(\mho+1)-1}\left[1-\frac{\rho}{\vartheta^2 B(1-\alpha)^{1-\vartheta}\left(\frac{1-\vartheta}{\vartheta}\right)^{1-\vartheta}\left(\frac{1-\tau_w}{1-s_d}\right)^{1-\vartheta}\left(\frac{y_0}{h_0}\right)^{1-\vartheta}}\right]}{1+\frac{(1-s_R)}{\theta\alpha}\left\{\frac{1}{(1-\tau_k)}\left[\sigma(\mho+1)+\frac{\rho[\sigma(\mho+1)-1]}{\vartheta^2 B(1-\alpha)^{1-\vartheta}\left(\frac{1-\vartheta}{\vartheta}\right)^{1-\vartheta}\left(\frac{1-\tau_w}{1-s_d}\right)^{1-\vartheta}\left(\frac{y_0}{h_0}\right)^{1-\vartheta}-\rho}\right]-\mho\right\}}$$

This expression shows that an increase in the R&D subsidy (s_R) has a positive impact on u_R^* while tax on capital income discourages investment in technology. The effects of the subsidy on education and the tax labor income are ambiguous. For a low level of α, the fraction u_R^* is reduced. This explains the market imperfection problem related to monopolistic competition. Thus, a powerful monopoly favors underinvestment in technology. To overcome this imperfection, the state can act through several policies to stimulate investment in R&D.

At the laissez-faire-equilibrium, the part of the time devoted to research and development is expressed by:

$$u_R^{LF} = \frac{1-\frac{\vartheta\mho}{\sigma(\mho+1)-1}\left[1-\frac{\rho}{\vartheta^2 B(1-\alpha)^{1-\vartheta}\left(\frac{1-\vartheta}{\vartheta}\right)^{1-\vartheta}\left(\frac{y_0}{h_0}\right)^{1-\vartheta}}\right]}{1+\frac{1}{\theta\alpha}\left[\sigma(\mho+1)-\mho+\frac{\rho[\sigma(\mho+1)-1]}{\vartheta^2 B(1-\alpha)^{1-\vartheta}\left(\frac{1-\vartheta}{\vartheta}\right)^{1-\vartheta}\left(\frac{y_0}{h_0}\right)^{1-\vartheta}-\rho}\right]}$$

The level $\hat{u}_R$ is the optimal value that we want to achieve. To detect the sources of economic and fiscal imperfections, we will start from the most preferred situation, for which the laissez-faire equilibrium solution coincides with the optimal value.

Theoretical analyzes show that the ratio $\left(\frac{\hat{u}_R}{u_R^{LF}}\right)$ equals the following quantity:

$$\frac{1-\frac{\vartheta\mho}{\sigma(\mho+1)-1}\left(1-\frac{\rho\left[\left(\frac{1-\vartheta}{\vartheta}\right)\left(1-\frac{D_{pub}}{D_{Totale}}\right)\right]^{\vartheta-1}}{\vartheta^2 B\left[(1-\alpha)\left(\frac{y_0}{h_0}\right)\right]^{1-\vartheta}}\right)\Big/ 1+\frac{1}{\theta}\left[\sigma(\mho+1)-\mho+\frac{\rho\mho}{\hat{g}_h}+\gamma\left(\frac{A_t}{A_{max}-A_t}\right)-\emptyset\right]}{1-\frac{\vartheta\mho}{\sigma(\mho+1)-1}\left(1-\frac{\rho\left(\frac{1-\vartheta}{\vartheta}\right)^{\vartheta-1}}{\vartheta^2 B\left[(1-\alpha)\left(\frac{y_0}{h_0}\right)\right]^{1-\vartheta}}\right)\Big/ 1+\frac{1}{\theta\alpha}\left[\sigma(\mho+1)-\mho+\frac{\rho[\sigma(\mho+1)-1]}{\vartheta^2 B\left[(1-\alpha)\left(\frac{1-\vartheta}{\vartheta}\right)\left(\frac{y_0}{h_0}\right)\right]^{1-\vartheta}-\rho}\right]}$$

This ratio is expressed in terms of the rate of growth of human capital, the share of public spending in the total expenditure on education and the distance to technology frontier indicated by the term $\left(\frac{A_t}{A_{sup}-A_t}\right)$. Analytically, an inequality between the two fractions (u_R^{LF} and $\hat{u}_R$) implies a situation of market inefficiency that requires the state's intervention through the appropriate policies to reach optimal values. For a reduced value of α, $\left(\frac{\hat{u}_R}{u_R^{LF}}\right)$ is high. Pushed to the extreme, this implies that the fraction u_R^{LF} is less than its optimal value. This implies that without state intervention, monopolistic competition can lead to underinvestment in technology. We note also that for a reduced value of the term $\left(\frac{A}{A_{sup}-A}\right)$ (a high technological gap), the quotient $\left(\frac{\hat{u}_R}{u_R^{LF}}\right)$ is high. This means that a country lagging behind the leader in technology is spending less on R&D. So, a big distance to technology frontier favors underinvestment in technology. Several important policies are required to overcome this type of imperfection. Economic openness, an increase in public spending on education in particular are the most favorable policies for the improvement of domestic capacity of innovating and absorbing foreign technologies. It is also important to note that the introduction of a well-harmonized and simplified tax system to further support innovation. More specifically, the state must choose the appropriate policy variables that allow the decentralized economy to achieve optimal growth.

Our theoretical analyzes identify that the first-best optimum can be decentralized by means of a tax on capital income at a constant rate $\left(\bar{\tau}_k = 1-\frac{1}{\alpha}\right)$, combined with an equality between the share of public spending in the total expenditure on education net of subsidy and the tax on labor income $\left(\frac{1-\bar{\tau}_w}{1-\bar{s}_d}\right) = \left(\frac{D_{priv}}{D_{Totale}}\right) \approx \left(\frac{\ell}{1+\ell}\right)$ and a time-varying subsidy to R&D. The following proposition determines the optimal subsidy $(\bar{s}_R)$ and its variation over time.

Proposition 2. *In the conditions of Proposition 1, the first-best optimal solution attainable by a central planner can be decentralized by means of a tax on capital income at a constant rate* $\left(\bar{\tau}_k = 1-\frac{1}{\alpha}\right)$, *combined with an equality between the share of public spending in the total expenditure on education net of subsidy and the tax on labor income* $\left(\frac{1-\bar{\tau}_w}{1-\bar{s}_d}\right) = \left(\frac{D_{priv}}{D_{Totale}}\right) \approx \left(\frac{\ell}{1+\ell}\right)$ *and a time-varying subsidy to R&D that evolves according to*

$$\dot{s}_R = g_A\left(\theta\alpha\frac{u_y}{u_R}\right) + (1-s_R)\left\{g_h\left(1-\frac{\vartheta}{u_h}\right) - r\left(1-\frac{2}{\alpha}\right) - g_A\left[\theta\frac{u_y}{u_R} - \gamma\left(\frac{A_t}{A_{max}-A_t}\right) + (\mho+1) + \emptyset\right]\right\}$$

and converges in the long-run to the optimal value

$$\bar{s}_R = 1 - \frac{\frac{\theta\alpha}{\sigma(1+\mho)-1}\left(\frac{\hat{u}_y}{\hat{u}_R}\right)\left[\vartheta^2 B(1-\alpha)^{1-\vartheta}\left(\frac{1-\vartheta}{\vartheta}\right)^{1-\vartheta}\left(\frac{\ell}{1+\ell}\right)^{1-\vartheta}\left(\frac{y_0}{h_0}\right)^{1-\vartheta} - \rho\right]}{\hat{r}\left(1-\frac{1}{\alpha}\right) + \hat{g}_A\left[\theta\frac{\hat{u}_y}{\hat{u}_R} - \gamma\left(\frac{A_t}{A_{max}-A_t}\right) + \emptyset\right]} < 1$$

which is financed by means of taxation.

The effect of externalities associated to R&D on the long-run value of the subsidy to R&D is stated in the following proposition.

V. Conclusion

This paper aims to characterize analytically the optimal dynamic fiscal policy in R&D-based endogenous growth model which incorporates domestic innovation, investment in education, distance to technology frontier and external technology spillovers through import of technologically advanced products and foreign direct investment as engines of growth. The model incorporates three sources of inefficiency: monopolistic competition in the intermediate-goods sector, duplication externalities and spillovers in R&D. To correct these imperfections, the intervention of the state by an effective fiscal policy is necessary. More specifically, the state must choose the appropriate policy variables that allow the decentralized economy to achieve sustainable optimal growth. To better understand this phenomenon, several theoretical analyzes were developed. To this end, we analyzed the efficient growth path that a benevolent social planner would implement. We provided conditions for the existence of a unique feasible optimal steady state with positive long-run growth. The optimal growth path can be decentralized by means of a tax on capital income at a constant rate combined with equality between the share of public spending in the total expenditure on education net of subsidy and the tax on labor income and a time-varying subsidy to R&D which addresses the duplication externalities and spillovers in R&D associated to the innovation process.

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