Fuzzy Linear Programming on Portfolio Optimization: Empirical Evidence from FTSE 100 Index

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Abstract- Portfolio is a list of securities that the investor has. The main objective of portfolio management is to maximize return while minimizing unsystematic risk. Firstly, fundamental definitions are given about theory of fuzzy logic and fuzzy logic approach is stated in this study. In the model of fuzzy logic price/earnings ratio and accumulation/distribution index which are added by the model that Werner improved. Taking all into consideration a new model is developed at the last part of this research.

Keywords: fuzzy linear programming, FTSE 100.

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I. INTRODUCTION

Investors are aiming to increase and protect their income in various ways by taking into account every condition that they encounter. For this reason, one way of applying this is using their incomes in financial markets. However, financial markets are affected by financial and social events, this causes a vague structure. To decide under this uncertainty is one of the hardest challenges for the investors.

Besides, investor’s knowledge and experience are very important during making the decision process. To use investors experience in the model will provide more realistic results. Fuzzy set theory is used to let uncertain conditions be modeled mathematically. (Mansur, 2002:1) Linear programming problems divided into three main components. Those components are decision variables, restricts and purpose function. In fuzzy linear programming, purpose function and purpose function coefficient are named as fuzzy target and represented by G. Fuzzy restricts are represented by C. For conclusion, decision for fuzzy target and fuzzy restricts is called fuzzy decision. Fuzzy decision is represented by D and μD(x) is the membership function of the fuzzy decision. Functions related to fuzzy targets is represented by μG(x), functions related to fuzzy restricts is represented by μC(x).

II. DECISION MAKING IN THE FUZZY ENVIRONMENT

Mathematical formulation of fuzzy set theory was created for the first time in 1965 by Zadeh. Zadeh introduced a way where uncertain conditions can be modeled mathematically. (Mansur, 2002:1) Linear programming problems divided into three main components. Those components are decision variables, restricts and purpose function. In fuzzy linear programming, purpose function and purpose function coefficient are named as fuzzy target and represented by G. Fuzzy restricts are represented by C. For conclusion, decision for fuzzy target and fuzzy restricts is called fuzzy decision. Fuzzy decision is represented by D and μD(x) is the membership function of the fuzzy decision. Functions related to fuzzy targets is represented by μG(x), functions related to fuzzy restricts is represented by μC(x).

Membership function related to targets is represented by μG(x) ∈ [0, 1] and valued from 0 to 1. If the membership function equals to 1 then target is fully achieved, if function equals to 0 target is not fully achieved. However, if membership function equals a value between 0 and 1 then target is partially achieved. Membership function related to restricts is represented as μC(x) ∈ [0, 1] and valued from 0 to 1. When membership function is equal to 0 then related restrict is not fully relevant, when equals to 1 then related restrict is fully relevant. When between 0 to 1, related restrict is partially relevant. Fuzzy decision is described as fuzzy target and fuzzy restricts are provided together. This is described as,

\[ D = G \land C \]  

Using equality membership functions in (1)

\[ \mu D(x) = \mu G(x) \land \mu C(x) = [\mu G(x), \mu C(x)] \]  

can be written (Terano et al, 1992). For more general description equalities in (1) ve (2) G1, G2, ..., Gn number of fuzzy targets and C1, C2, ..., Cm m number of fuzzy restricts,
\[ D = G_1, G_2, \ldots, G_n, C_1, C_2, \ldots, C_m \]  

with membership functions (Bellman and Zadeh, 1970: 141-164).

\[ \mu D(x) = \min \{ \mu G_1(x), \mu G_2(x), \ldots, \mu G_n(x), \mu C_1(x), \mu C_2(x), \ldots, \mu C_m(x) \} \]

In order to achieve optimum decision in the problem, the highest degree of the element in the fuzzy decision set should be determined. This is calculated as (Terano et al. 1992).

\[ \mu D(x) = \max \mu D(x) \]

The equality in (5) is known as max-min processor. Max-min processor is a reliable method to choose the best solution between the worst cases. Extensively Max-min processor is written as,

\[ \max \mu D(x) = \max (\min (\mu G(x), \mu C(x))) \]

III. Fuzzy Linear Programming and Portfolio Analysis

Investors are aiming to increase and protect their income in various ways by taking into account every condition that they encounter. For this reason, one way of applying this is using their incomes in financial markets. However, financial markets are affected by financial and social events, this causes a vague structure. To decide under this uncertainty is one of the hardest challenges for the investors. Uncertainty in this environment brings lots of risks parameters for the investors. Investors are trying to reduce risk factors into minimum by using different instruments for their assets. By creating portfolio and managing it, risk is already reduced. Because, risk of the portfolio as a whole is smaller than risks that every share possesses individually. But, over diversification can be harmful while creating the portfolio. While doing over diversification, low-performance investment instruments are included in the portfolio. Also, it can be harder to provide information about investment tools when the number is increased. Generally, portfolio is a new entity, which has measurable qualities in relation with together to fulfill certain purposes (Ceylan and Korkmaz, 1998). Portfolio is a pool in where at least two instruments are in it in order to reduce risk and provide the highest income due to that risk (Ercan ve Ban, 2005).

Markowitz’s modern portfolio approach put forward in 1952 by at least risk level needed to reach the targeted level of investor returns and begin to determine the structure of the portfolio risk level (Ulucan, 2004). Although it is theoretically appropriate, Markowitz portfolio optimization model is not preferred in practice for especially large-scale portfolios. The most important reason behind the practical usefulness of the Markowitz model poses challenges emerging in the solution of quadratic programming problems with large-scale covariance matrix.


As a model portfolio of functional formulation requires the return of the shares that make up the portfolio and estimation of the distribution of the risk. Information on the selected shares during the time interval, the return and risk distribution of the portfolio is random therefore managers of the portfolios should have reviews regarding shares provides great importance.

These different interpretations by different portfolio managers can be caused from the same set of information. Having different interpretations of the portfolio by the managers will be transferred to the portfolio models created with use of fuzzy set theory.


In this study, based on recommended model by Konno and Yamazaki (1991: 515-531) Fang and others (2005:879-893) will try to create optimum portfolio. This model is explained below.

Here, \( pM0 \) (expected fuzzy income amount), is in the closed interval of \( \tau \), tolerance value known amount of expected \( [pM0, pM0+\tau] \). \( pM0+\tau \), is determined by decision maker as an upper value of expected income.

\[ \min w(x) = \frac{\sum_{t=1}^{T} y_t}{T} \]

\[ y_t + \sum_{j=1}^{n} a_{tj} x_j \geq 0, \quad t = 1,2, \ldots, T \]

\[ y_t - \sum_{j=1}^{n} a_{tj} x_j \geq 0, \quad t = 1,2, \ldots, T \]
\[ \sum_{j=1}^{n} r_j x_j \geq \rho M_0 + \tau \]
\[ 0 \leq x_j \leq 1, \ j = 1,2, ..., n \]
\[ \sum_{j=1}^{n} x_j = 1 \]

Here, \( \rho M_0 \) (expected fuzzy income amount), is in the closed interval of \( \tau \), tolerance value known amount of expected \([\rho M_0, \rho M_0 + \tau] \). \( \rho M_0 + \tau \), is determined by decision maker as a upper value of expected income.

This model can be used to determine how much to invest in to different stocks by using \( \alpha [0, 1] \) for different levels of expectation. Besides, decision makers at this level can determine target income and risk values at specified level.

However, the main purpose of this model is to achieve an optimum solution from a variety of combinations of return and risks are not fully adequate. Werners have suggested that the objective function due to blurred and fuzzy inequality constraints sources may also be fuzzy. As in Verdegay’s approach, every fuzzy source tolerance is assumed to be known. In order to apply Werner’s approach to the model is solved for \( \rho M_0 \) (\( \alpha = 0 \)) and \( \rho M_0 + \tau \) (\( \alpha = 1 \)) expected income and function values are found as \( Z_0 \) and \( Z_1 \) (minimized risk values).

As the expected income value in the model is increased, the minimized risk value will also increase and therefore \( Z_1 > Z_0 \). As the investors are sensitive to risk, when risk is increased, satisfaction will decrease. When the membership functions are introduced in the linear programming model, fuzzy target DP model becomes standard DP model below:

\[ Maks. \alpha \]
\[ \sum_{t=1}^{T} \frac{y_t}{T} + \alpha (Z^1 - Z^0) \leq Z^1 \]
\[ y_t + \sum_{j=1}^{n} a_{jt} x_j \geq \rho, \ t = 1,2, ..., T \]
\[ y_t - \sum_{j=1}^{n} a_{jt} x_j \geq 0, \ t = 1,2, ..., T \]
\[ \sum_{j=1}^{n} r_j - \alpha \tau \geq \rho M_0 \]
\[ 0 \leq x_j \leq 1, \ j = 1,2, ..., n \]
\[ \sum_{j=1}^{n} x_j = M_0 \]

Here, \( \tau \) is the tolerance value of expected rate of return.

After that, parametric equation is solved accordingly for expected incomes \( \rho M_0 \) (\( \alpha = 0 \)) and \( \rho M_0 + \tau \) (\( \alpha = 1 \)), by doing this \( Z_0 \) and \( Z_1 \) (minimized risk values) target function values are found.

\[ \min w(x) = \frac{\sum_{t=1}^{T} y_t}{T} \]
\[ y_t + \sum_{j=1}^{n} a_{jt} x_j \geq 0, \ t = 1,2, ..., T \]
\[ y_t - \sum_{j=1}^{n} a_{jt} x_j \geq 0, \ t = 1,2, ..., T \]
\[ \sum_{j=1}^{n} r_j \geq \rho M_0 + \tau \]
\[ 0 \leq x_j \leq 1, \ j = 1,2, ..., n \]
\[ \sum_{j=1}^{n} x_j = M_0 \]

By solving with this model, \( Z_0 = 0.0080 \) and \( Z_1 = 0.0113 \) values are found.

After finding \( Z_0 \) and \( Z_1 \) values, target membership function, when \( \alpha = 0 \) is \( Z_0 \) and when \( \alpha = 1 \) \( Z_1 \) values are used to determine target membership function like below.

\[ \mu_z(x) = \begin{cases} 
1, & Z < Z_0 \\
1 - \frac{Z - Z_0}{Z_1 - Z_0}, & Z_0 \leq Z \leq Z_1 \\
0, & Z > Z_1 
\end{cases} \]

\[ \mu_z(x) = \begin{cases} 
1, & Z < Z_0 \\
1 - \frac{Z - 0.0080}{0.0033}, & 0.0080 \leq Z \leq 0.0113 \\
0, & Z > 0.0113 
\end{cases} \]

By putting membership functions into their places, fuzzy target and sourced DP model becomes standard DP model.

\[ \text{Max } \alpha \]
\[ \sum_{t=1}^{T} y_t + \alpha (Z_1 - Z_0) \leq Z_1 \]
\[ y_t + \sum_{j=1}^{n} a_{jt} x_j \geq 0 \]
\[ y_t - \sum_{j=1}^{n} a_{jt} x_j \geq 0 \]
\[ \sum_{j=1}^{n} r_j x_j - \alpha \tau \geq \rho M_0 \]
\[ \sum_{j=1}^{n} x_j = M_0 \]
\[ 0 \leq x_j \leq 1 \]
\[ \alpha \in [0,1] \]

By solving this model, \( \alpha = 0.51 \) is found. Minimum risk ratio related to this \( \alpha \) value is found using membership function as below:

\[ \mu_z(x) = \alpha = 1 - \frac{z - 0.0080}{0.0033} \]
\[ 0.51 = 1 - \frac{z - 0.0080}{0.0033} \]
\[ z = 0.009617 \]

With \( \alpha = 0.51 \) satisfaction level minimized risk ratio \( z \) is calculated as approximately % 9.6. In this satisfaction level expected rate of return is;

\[ \text{Expected Return} = \rho M_0 + \alpha \tau \]
\[ = 0.000325 + 0.51 \times 0.00076 \]
\[ = 0.000325 + 0.0003876 \]
\[ = 0.0007126 \]

With \( \alpha = 0.51 \) satisfaction level, with taking %9.6 as risk, expected rate of return is around %0.7. The table below shows that after solving the recommended model, stocks which should be present in the portfolio and the amount of stocks in the portfolio (%).

**Table 1 : Weights of Shares in the Target Portfolio**

<table>
<thead>
<tr>
<th>( x_j )</th>
<th>Shares</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_1 )</td>
<td>ANGLO AMERICAN</td>
<td>0</td>
</tr>
<tr>
<td>( x_2 )</td>
<td>BRITISH AMERICAN TOBACCO</td>
<td>0.6065</td>
</tr>
<tr>
<td>( x_3 )</td>
<td>CARNIVAL</td>
<td>0</td>
</tr>
<tr>
<td>( x_4 )</td>
<td>DIAGEO</td>
<td>0</td>
</tr>
<tr>
<td>( x_5 )</td>
<td>EXPERIAN</td>
<td>0</td>
</tr>
<tr>
<td>( x_6 )</td>
<td>FRESNILLO</td>
<td>0</td>
</tr>
<tr>
<td>( x_7 )</td>
<td>GLAXOSMITHKLINE</td>
<td>0</td>
</tr>
<tr>
<td>( x_8 )</td>
<td>HIKMA PHARMACEUTICALS</td>
<td>0.3935</td>
</tr>
</tbody>
</table>
In the portfolio created using the values, %60.65 of BRITISH AMERICAN TOBACCO stocks and %39.35 of HIKMA PHARMACEUTICALS stocks should be presented.

V. Conclusion

Behind the portfolio concept, idea of risk minimization lies. For this reason, in order to invest the assets into only one instrument, it is beneficial to invest a portfolio which consists of more than one instrument. This diversification should be done by comparing the stocks in the portfolio or in the sector they are in. By doing this, expected rate of return could be achieved easily. The study which Markowitz conducted in 1952 created new horizons for the investors. In the meantime, new assumptions and approaches are created after Markowitz’s work. Linear programming model by Konno-Yamazaki, which is an approach to this model, was fuzzed by Werners and other researchers. In this study, Werner’s model using fuzzy linear programming for portfolio optimization is taken as a basis.

This model is now examining the situation and the past performance of stocks in the sector, which is one of the main methods of analysis Price / Earnings ratio of technical analysis and collection - distribution index is created as a new model by adding constraints. By solving the proposed model by economic package program, portfolio is created. In this portfolio, there should be BRITISH AMERICAN TOBACCO stock by %60.65 and HIKMA PHARMACEUTICALS stocks by %39.35. This portfolio is expected to have a rate of return of % 0.7 with %9.6 risk.

References Références Referencias