# Strengthening Libor & Rate-Setting Processes: Recommendations for Policymakers

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## Abstract

The authors present two different models to better understand and analyze the recent Libor scandal. These models can be applied generally to other rate-setting processes and can be used by policymakers and regulators to effectively monitor benchmarks for manipulation

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*Index terms*— monitor benchmarks for manipulation

# 1 INTRODUCTION

he authors present two different models to better understand and analyze the recent Libor scandal. These models can be applied generally to other rate-setting processes and can be used by policymakers and regulators to effectively monitor benchmarks for manipulation.

# 2 a) Basics of Libor

Libor was administered by the British Bankers Association (BBA) with Thomson Reuters as the calculation agent. For ten different currencies with 15 maturities each -a total of 150 rates every business day -contributor banks would submit rates giving an indication of the average rate at which they can obtain unsecured funding in the London interbank market for a given period, in a given currency. Every contributor bank is asked to base their Libor submissions on the following question: "At what rate could you borrow funds, were you to do so by asking for and then accepting inter-bank offers in a reasonable market size just prior to 11 am?"

# 3 b) Manipulation of Libor

It should be noted that the submissions of banking institutions are not necessarily based on actual transactions. In fact, a bank is not legally required to lend to other banks at the rate of its submission. So while banks were not supposed to have a vested interest in their reported rate, manipulation was inevitable.

There are two general categories of Libor manipulation -perception and trading. Perceptionbased manipulation is an intentional under-reporting of Libor to project an image of stability and health. Tradingbased manipulation is the intentional under-reporting, over-reporting or holding constant of Libor to benefit trading positions or trader compensation. Internal emails uncovered from a Barclays trader revealed that even a basis point (.01%) drop in the Libor rate could create a few million dollars in gains for his positions.

A basic understanding of Libor is enough for the purpose of this paper, but a more in-depth understanding of Libor and the Libor fixing scandal can be found in the Statement of Facts that was part of the non-prosecution agreement between the United States Department of Justice and Barclays. * II.

# 4 GAME THEORY APPROACH

A bank estimates today's (unbiased) Libor submission rate to be 1%. The bank wants today's Libor to be set lower than 1%. What rate should they submit to increase the likelihood that Libor will be set in its favor?

The bank already knows the following setup of the game:

? Out of a total of 18 submitted rates, the highest four and lowest four submissions are excluded from the average.

42    ? Significant deviations from the trimmed average will attract the unwanted attention of regulators.

43    ? The estimated standard deviation of the reported rates from other agents will equal $1/10 = 10$ bps, i.e. the
44 typical bid/offer (Libid/Libor) spread for borrowing and lending between banks.

45    ? Assuming the bank's rate falls within the 10 averaged rates, the maximum contribution of the submitted
46 rate x would be $(1\% - x) / 10$. Under these conditions, a bank could adjust its submission within a reasonable
47 range of 12.5bps, give or take a few basis points.

# 5   a) Formulation of Unbiased Conditions

49 We can mathematically represent the Libor process as follows:?? = ? ? ?? ?? ???? ??=??+1 ? /(?? ? ?? ? ?)

50    Where L: fair (unbiased) Libor rate $S_i$ : rate submission of participant i n: total number of participants
51 surveyed k: number of lowest submissions discarded h: number of highest submissions discarded

52    We assume there is no collusion among participants and every submission is independent. All submissions are
53 samples from the same distribution with density function f(x) and cumulative distribution function F(x).

54    If a fair estimate of Libor is 1%, the distribution can be uniform [0.9%, 1.1%] with a mean of 1% and standard
55 deviation of 12.9bps.

# 6   i. Example

57 Assume the 18 participants of the Libor process provide the following submissions ordered lowest to highest: If
58 k = 4 and h = 4, then:?? = ? ? ?? ?? 18?4 ??=4+1 ? /(18 ? 4 ? 4) = ?? ?? ?? 14 ??=5 ? /10 = 1 b)
59 Introduction of Manipulation

60    Now assume a participant has an interest in Libor set under the fair rate, L. This participant passes artificially
61 depressed submission X, such that $X_i < S_i$ . Keep in mind that an $S_i$ that is too low would have no impact on
62 L.

63    Our objective would be for X to be the (k + 1) lowest number, the lowest number that would be included in
64 (not discarded from) the average calculation. Our contribution to the calculation of the rate in that case would
65 be $(L - x) / (n - k - h)$.

66    To derive our expected benefit, we start by noting that of the (n -1) remaining participants there are (n -1)
67 $C_k = (n-1)! / (k! * (n-k-1))!$ ways of choosing the lowest k estimates. The probability that all of these
68 estimates will give an estimate less than x is $F(x)^k$ . Of the (n -k -1) participants, the probability of submitting
69 an estimate higher than x is $[1 - F(x)]^{(n-k-1)}$ . Our expected benefit from submitting rate x in place of our
70 original estimate L is $x * \{(n-1)! / (k! * (n-k-1)!)\} * \{F(x)^k * [1 - F(x)]^{(n-k-1)}\}$. Our underlying goal would
71 be to maximize the result of this equation. This maximization can be carried out either numerically, or by taking
72 the logarithm of the above expression and putting its derivative equal to zero.

73    To derive the zero of the equation we maximize the logarithm of the equation $\ln(x) + k * \ln(F(x)) + (n-k-1)$
74 $* \ln(1 - F(x))$. To solve, we take the derivative of this new equation and set it equal to zero:$(1 / x) + [k * f(x) /$
75 $F(x)] - [(n-k-1) * f(x) / (1 - F(x))] = 0$. Alternatively, we can formulate the equation as: $[F(x) * (1 - F(x))] + [k$
76 $* x * f(x) * (1 - F(x))] - [(n-k-1) * x * f(x) * F(x)] = 0$. i. Example

77    For an intuitive understanding of this model for manipulation let us apply it to Libor setting process. The
78 Libor process is comprised of 18 participant financial institutions. The 4 lowest and 4 highest submissions are
79 discarded. So, n = 18; k = 4; h = 4; (n -k -h) = 10. We assume that the "fair and unbiased" estimate of Libor
80 to be 0.5% and the participants provide independent estimates from the uniform [0,1] distribution.

81    Based on this setup, an institution with an interest in a lower setting of Libor would want to provide the 5
82 th lowest estimate (k + 1 = 4 + 1 = 5). To optimize their benefit this institution would need to submit an
83 estimate x such that the equation $\ln(x) + 4 * \ln(x) + 13 * \ln(1 - x) = 5 * \ln(x) + 13 * \ln(1 - x)$ is maximized.
84 The derivative of this equation results in: $(5 / x) - [13 / (1 - x)] = 0$. Solving for x, we arrive at x = 5 / 18 =
85 0.278%. This institution should submit an estimate of 0.278% to optimize its benefit. In other words, this value
86 balances the risk of being discarded as a low bid and excluded from the averaging process and the chance of being
87 included in the averaging process and pulling the average down in the banks favor i.e. maximizing our expected
88 contribution to a lower setting of Libor.

# 7   c) Implication For Policymakers

90 Now we introduce uncertainty into our formulation. Suppose that of the 16 participant rate submissions the k
91 lowest and (8 -k) highest are discarded. Furthermore, k is unknown to the participants beforehand. An institution
92 with an interest in a lower setting of Libor would want to provide the (k + 1) th lowest estimate. Assuming the
93 same uniform distribution for submissions the optimal estimate would minimize $\ln(x) + k * \ln(x) + (16-k-1) *$
94 $\ln(1-x) = (k+1) * \ln(x) + (16-k-1) * \ln(1-x)$. The derivative would equal $[(k+1) / x] - [(16-k-1) / (1-x)]$
95 $= 0$. Solving for x we arrive at x = (k + 1) / 16.

96    With this equation x = (k + 1) / 16 we arrive at significant policy implication for reducing Libor-setting
97 (or any similar benchmark-setting) manipulation: a ratesetting body should both randomize and withhold the
98 number of top and bottom submissions to be discarded. In the context of Libor for example, the BBA should
99 have discarded the top 4 and bottom 4 one day, the top 2 and bottom 6 the next day, top 5 and bottom 2 the
100 next day and so on all while not disclosing these values until ex post facto.

In hindsight an unknown k should be intuitive—the more unknowns added to an equation the more difficult it becomes to solve. The increased uncertainty increases the potential risk of drawing unwanted attention to a submission (assuming regulators are paying attention).

# 8 III.

# 9 EMPIRICAL BAYESIAN APPROACH

In our game theory approach it was assumed that there was no cooperation or collusion among participants. But what if this is not the case? How could collusion be detected?

An empirical Bayes model can provide a framework to answer this question. Libor submissions are seen as a repeated game. A trader, or policymaker, will begin with the belief in the integrity of the market, i.e. Libor submission process. As the trader observes repeated Libor submissions and identifies potential bias, he will adjust this prior belief in the integrity of the market. His subsequent posterior beliefs continue to erode his belief in the integrity of the market.

For example, suppose a junior trader faces pressure from her managers to submit Libor estimates that benefit their firm. Her managers rationalize and justify their request by claiming that all participating institutions manipulate their submissions and that "if you can't beat 'em, join 'em". Nonetheless, the trader is uneasy with her managers' request.

Assume the trader's hypothesis $H_m$ is that the Libor rate is not manipulated, and is fair and the process is policed by regulators to ensure an unbiased estimate of LIBOR. Her confidence in the integrity of the market can be represented as the subjective probability of the truthfulness of her hypothesis of market integrity as $p(H_n) = 99.99\%$. Let's call this the Prior.

Her idealism starts to fade as she observes evidence E of biased Libor submissions from other institutions. Her growing suspicion can be represented as the posterior probability of her faith in the market updated in light of new evidence, or $p(H_n|E)$. Let's call this the Posterior probability. What is the probability of her belief in the integrity of the market given E?

According to the Bayes rule this is $p(H_n|E) = [p(E|H_n) * p(H_n)] / p(E)$

The Posterior probability equals the $p(E|H_n)$—chance of observing biased submissions if the market is unbiased and the biased is purely due to chance—multiplied by $p(H_n)$ -the Prior probability -divided by the market's bias or unbias. Note that $P(E) = P(\text{observation of biased submissions}) = P(\text{observation of biased submissions} | \text{market is biased}) * P(\text{market is biased}) + P(\text{observation of biased submissions} | \text{market is unbiased}) * P(\text{market is unbiased})$.

According to the above formula if $[p(E|H_n) / p(E)] <1$ then $p(H_n|E) < p(H_n)]$. In other words, each biased Libor submission weakens the trader's belief in a market with integrity, $p(H_n)$ before observing the biased submission.

It's worth repeating our conclusion: $p(E) > p(E|H_n)$ or $[p(E|H_n) / p(E)] < 1$. This implies $p(H_n|E) < p(H_n)$. [1] [2]

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[1] * http://www.justice.gov/iso/opa/resources/9312012710173426365941. pdf