A Real Options Approach to Contractual Agreements and Value Flexibility

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Abstract - Contracts are usually analyzed in the light of the reduction of transaction costs that they may ensure. But this disregards the advantages of strategic flexibility in business relations. In this paper we consider a model of provider-client relation and see how flexibility in the contract (seen as a combination of a put and a call option) ensures a higher payoff to the involved parties.

Keywords: contracts, transaction costs, real options.

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Keywords: contracts, transaction costs, real options.

1. Introduction

A firm can be seen, abstractly, as a portfolio of agreements with outside partners, yielding costs and benefits. The firm lowers its exposition to uncertainty by means of rigid contracts, but their positive effects are overshadowed by the corresponding loss of strategic flexibility: expected benefits from future business opportunities can be lost due to the binding obligations that force their rejection.

The advantages of contracts, particularly those intended to protect investments in specific assets has been predicated in terms of the reduction of transaction costs [21] [14]. The protection is obtained through low-yield, transaction specific investments, covering the risks derived from three possible sources: malicious behaviors of other agents, contingencies of the market or changes in technology. Most of this literature treats only static models, focusing on behavioral risks. But this approach disregards the maneuvering possibilities of benefitting the strategic flexibility provided by the dynamics of potential market opportunities.

This paper intends to suggest ways to enhance contracts by means of Real Options. We derive decision-making models in which a balance is reached between protection (with its concomitant loss of flexibility) and openness to business opportunities. Our approach complements the literature on strategic tradeoffs between flexibility and contracts [2] [3][4] [14]. A brief discussion of these references can help to put into context our own take on the problem. [2] and [3] explore the consequences of contracts over corporate governance, claiming that real options are the right tools for the design of corporate structures. [14], in turn, develops a binomial options model that trades-off agreement and strategic flexibility. Following this lead we will consider a binomial model of valuation of exchange options on provision contracts. Considering complete information firm-client games, we will see that an adequate balance between profits and punishments allow to support, in Nash equilibria, both the enforcement of contracts and the adequate means to breach them when better alternatives become available.

The plan of the paper is as follows. Section 2 compares the literature on transaction costs with the recommendations of real options analysis, in order to see which aspects should be taken from each of these two approaches. Section 3 develops an example in which a stepwise analysis shows that an adequate degree of flexibility can be good for both parties in a contract. Section 4 draws the conclusions of the exercise and concludes.

II. Transactions Cost Analysis vs. Real Options

Transaction Cost Analysis (TCA) seeks to design efficient mechanisms, minimizing transaction costs [20] [21]. The ensuing contracts are intended to protect economic relations among agents. Some of their associated costs are due to the transactions leading to agreements. The main sources of transaction costs considered in this literature are:

- Bounded rationality: agents are assumed to have only limited capacity of acquiring and processing information, restricting their self-interested decision-making abilities.
- Generalized uncertainty: while the intentional behavior of other agents is its main source, the business context in which the firm acts (the economy, the production sector to which it belongs, the technology, etc.) adds more uncertainty to decision-making.
- Specificity of transactions: non-specific liquid assets provide efficient mechanisms supporting exit or sale options. Exiting is more costly for specialized and less liquid assets that demand extra provisos for the protection of investments.

Of these, the existence of investments in specific assets and the pervasiveness of uncertainty are perhaps the most important factors. The former involves assets satisfying only specific exchange relations, having low recovery value outside those relations. They are risky in the sense that their excess value can be appropriated by the business counterparts of the firm [10]. Transactional uncertainty, on the other hand, arises...
from unforeseen contingencies. For instance, in the main application of this paper, namely provider-client relations, it amounts to the difficulty of predicting the volume that will be actually demanded to the supplier, due to the volatility of the market in which the client operates. The ensuing renegotiations induce extra costs to be accounted for in the contracts.

While exchange relations could arise without any previous agreement, TCA prescribes vertical integration as a way of minimizing transaction costs, providing coverage against uncertainty. This allows those costs by substituting the market by agreed-on buyer-supplier actions in coordinated fashion [8]. The advantages of vertical integration are evident in stable business contexts, where transaction dynamics and the possibility of new opportunities can be disregarded [14]. If the latter is not the case it becomes necessary to allow a degree of strategic flexibility.

A possible way of achieving strategic flexibility is by means of options and concomitant incentives to respect (or break) contracts. The former bound the responses to the dynamics of the business context, amplifying gains or fixing a lowest value to losses [7]. The theoretical framework in which real options (RO) are analyzed arises with the Black-Scholes-Merton model [5] [11]. While financial problems are mostly analyzed through continuous-time models ([22]), the valuation of strategic flexibility has been carried out in discrete-time frameworks ([19] [1] [12]), which are variants of the classical binomial model [6]. The use of this models allow firms to increase gains and cut down losses [16].

To further compare the prescriptions of TCA and RO notice the both consider sequential decision-making under uncertainty ([21] [18]) as well as the irreversibility and specificity of investments [7] [17]. But they differ in their underlying notions of rationality and their effects on how they handle uncertainty: while RO assumes full rationality and information processing capacity, TCA considers, as said, boundedly rational agents. In the latter case, contracts are incomplete, since not all possible states are conceivable and consequently incorporated into contracts. But these differences allow complementarities between the two approaches. On one hand, TCA focuses on the protection against unexpected behaviors, reducing flexibility, while RO, on the other, provides coverage against environment uncertainty yielding more strategic alternatives. Our approach will take the best from both approaches.

III. REAL OPTIONS AND GAME-THEORETICAL CONSIDERATIONS: PROVISION CONTRACTS

We will develop a model featuring all the aspects we intend to capture. Consider a input supply contract for which we will determine the benefits of the preservation of assets compared against the loss of flexibility. More precisely, we will contrast the current value of the contract with that of the option of changing to a potential alternative client. Since transactions are carried out in discrete time we will use a binomial approach for the stochastic model of uncertainty.

On the other hand, the agreement on payments and punishments for breakups are determined as Nash equilibria in complete and perfect information games.

We break our analysis in three: Case A assumes a technologically stable environment, determining the value of the contract and the cost of breakup. Uncertainty of demand is obtained in a binomial model. Case B adds an option of changing to a new contract. The comparison of the values of the old contract and the option yields costs and benefits of renouncing to the former. Finally, Case C introduces further flexibility into the contract, defining:

1) The minimal price to be agreed on with the new client, taking into account the costs involved in breaking up the original contract.
2) The optimal amounts to be supplied to both the old and new client, assuming that the prices and the plant capacity are fixed.

a) Case A: Agreement in a Stable Environment

Consider a supplier \( P \) providing some input to a client \( C \) who uses it to make some final product. To provide this input, a previous investment \( t \) in period \( t_i \) is necessary, yielding benefits starting in \( t_i \). This investment is highly specific and irreversible. It cannot be deferred and has no certain recovery value. The parties agree to carry out transactions for three periods, negotiating prices ex ante. Suppose the agreed on unit price \( p_c \) of a unit of the final product in \( t_i \) such that \( p_c > c_o \) where the operation cost is \( c_o \) per unit. Thus, the benefits for \( P \) at any period \( t_i \), \( t_i \) and \( \beta \) are \( p_c - c_o \). The market value of the product \( v_c \) is such that \( v_c > p_c \). The demand of input is uncertain, but can modeled as a binomial process where the initial demand of \( C \)'s product is \( q_i \). Two states are possible: a "good" one in which demand grows by a factor of \( u > 0 \), and a "bad" one in which the demand falls by a factor \( d > 0 \). The risk-less rate of interest is \( r \) per period. The risk-neutral probabilities are thus [6]:

\[
\rho = \frac{(1 + d - d)}{u - d} \quad \text{and} \quad 1 - \rho. \quad 1
\]

\( \rho \) This means that in time the demand evolves as follows:

\[
\begin{array}{cccc}
1 & 1 & 1 & 1 \\
q_c & q_c u & q_c u d & q_c u d d \\
q_c & q_c u & q_c u d & q_c u d d \\
q_c & q_c u & q_c u d & q_c u d d \\
\end{array}
\]

and the expected demand at period \( t \), \( \mathbb{E}[q^t] = \sum_{k=0}^{t} \frac{u}{k!} (r) \sum_{k=0}^{t} \frac{d}{k!} (t-k) p^k \)

\((1 - p)^{t-k} q_c u d^k d^{t-k} \)
The benefits of a binding contract have to be compared to the results of only agreeing on an initial price without a long term commitment. Assume that both parties agree on an initial price \( \bar{p}_c \) per unit and that in the next three periods each party will try to capture the excess benefits, based on their respective bargaining powers. Furthermore, assume that while \( v_c > c_o, \bar{p}_c > \frac{v_c + c_o}{2} \), i.e. \( pc \) is in the right half of the interval \([c_o, v_c]\). At \( t_1 \), \( P \) worries that \( C \), being his only customer, will try to get hold of the current value of ‘s own benefits, offering a price \( p_c = c_o \). On the other hand, \( C \) fears that the monopoly power of \( P \) will allow the latter to fix a higher price \( \bar{p}_c > v_c \). Without external providers or customers, both are in a bilateral monopoly situation. In this case \( P \) and \( C \) might agree on keeping the pre-agreed price \( \bar{p}_c \), or might agree in deviating, sharing the excess benefits in proportion to their bargaining power, which we assume is the same for both. Alternatively, one of the parties may try to impose its terms on the other. In the next periods the parties repeat the game, either agreeing to keep the original price or engaging in another round of bargaining. The following matrix exhibits the strategies and the payoffs the players would get if the parties follow suit:

<table>
<thead>
<tr>
<th>( P/C )</th>
<th>Keep</th>
<th>Deviate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Keep ( \bar{p}_c - c_o, v_c - \bar{p}_c )</td>
<td>( \frac{v_c - c_o}{2} )</td>
<td>( \frac{v_c - c_o}{2} )</td>
</tr>
<tr>
<td>Deviate ( v_c - c_o, \bar{p}_c - v_c )</td>
<td>( \frac{v_c - c_o}{2} )</td>
<td>( \frac{v_c - c_o}{2} )</td>
</tr>
</tbody>
</table>

Since \( v_c - c_o > 0, \bar{p}_c - c_o \), for \( P \) and both \( v_c - c_o > \bar{p}_c - c_o \), for \( P \), the only Nash equilibrium (in dominant strategies) is that both players Deviate. The following matrix shows the corresponding asked prices at all four possible outcomes:

<table>
<thead>
<tr>
<th>( P/C )</th>
<th>Keep</th>
<th>Deviate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Keep ( \bar{p}_c )</td>
<td>( \bar{p}_c )</td>
<td>( \bar{p}_c )</td>
</tr>
<tr>
<td>Deviate</td>
<td>( v_c )</td>
<td>( v_c )</td>
</tr>
</tbody>
</table>

To analyze the outcome in the repeated game consider the discounted cashflows of the agents, based on the net present value of both agents, given C’s demand constraints:

\[
NPV_P(s_p, s_C) = -I + \sum_{t=1}^{3} \frac{1}{(1+r)^t} E(q)_t B_P(s_p, s_C) \\
NPV_C(s_p, s_C) = \sum_{t=1}^{3} \frac{1}{(1+r)^t} E(q)_t B_C(s_p, s_C)
\]

Where \( s_p, s_c \in \{\text{Keep, Deviate}\} \) while \( B_P(s_p, s_c) \) and \( B_C(s_p, s_c) \) are the instantaneous unit benefits for \( P \) and \( C \) respectively, when they choose \( s_p \) and \( s_c \) at period \( t \).

3 Notice that disagreements lead to break-ups of the contract, since a player that chooses to keep the original price would not accept the terms of the other player.

There are of course many cases that can be analyzed. But recall that unilateral deviation leads to the breakup of the contract and zero benefits for both parties. So we will focus on the cases in which either both agree in keeping \( 2p_p \) or both deviate, sharing in equal parts the excess benefits. We have that \( NPV_C(Deviate, Deviate) < NPV_C(Keep, Keep) \) and we assume that \( 1/5 \) is less than the discounted flow of benefits at least at \( t \) (Keep, Keep).

But then, if \( C \) agrees on keeping the original price, \( P \) has incentives to deviate. On the other hand, \( NPV_C \) is larger in the stage Nash equilibrium than when both parties agree on keeping \( \bar{p}_c \) and thus has no incentive to agreeing on that.

The traditional way of addressing this in a repetition is by means of a trigger strategy, which punishes any move that goes against a desired result [13]. Consider the case in which the original contract is to be enforced, i.e. (Keep, Keep). We need to establish the appropriate punishments for both parties, \( \{M_p, M_C\} \) that make agreeing the dominant strategy in the game and thus keep the price at \( \bar{p}_c \). Consider the benefits at each period \( t \) when these penalties are enforced:

<table>
<thead>
<tr>
<th>( P/C )</th>
<th>Keep</th>
<th>Deviate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Keep ( \bar{p}_c - c_o, v_c - \bar{p}_c )</td>
<td>( \frac{v_c - c_o}{2} - M_p )</td>
<td>( \frac{v_c - c_o}{2} - M_p )</td>
</tr>
<tr>
<td>Deviate ( v_c - c_o - M_P, v_c )</td>
<td>( \frac{v_c - c_o}{2} - M_p )</td>
<td>( \frac{v_c - c_o}{2} - M_p )</td>
</tr>
</tbody>
</table>

To ensure that (Keep, Keep) is the only Nash equilibrium, it suffices to fix \( M_p \) and \( M_C \) to be larger than \( v_c - c_o \). That is, larger than the excess benefits of the transaction.

b) Case B: Agreement on the provision in a dynamic environment

Let us assume now that the technological environment is dynamic, due to the entrance of new agents, in this case potential customers of \( P \). As before, assume an agreement between \( P \) and \( C \) at \( t_0 \). But at \( t_1 \) \( P \) finds a potential new customer \( Z \). Assuming that \( P \) has a limited capacity of provision, she has to decide on either to respect the original agreement or to break it and make an agreement with \( Z \). This can be seen as if \( P \) had an option that combines a long call and a put position. Breaking the agreement with \( C \) is like enabling an European sale option, while starting a new relation with \( Z \) is like activating an European buy option (call). The latter is exerted at \( t_2 \), at which point \( C \) is dropped by \( P \). The put has null exercise price while the new contract demands a marginal investment in the production facilities of \( Z \) at \( t_2 \). Given \( Z \)’s initial demand

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3 Due to the condition on \( \bar{p}_c \).
4 Since otherwise it wouldn’t be rational to sign the initial contract.
5 We assume that if only one party deviates, the penalty it pays is transferred to the other.
$q_{r1}$ its uncertainty is described also by a binomial process with rates $u_c$ and $d_z$.

With an agreed on price $\bar{p}_z$, given the operational cost $c_z$ the instantaneous profit of $P$ is $\bar{p}_z - c_z$. On the other hand, the value of one unit for $Z$ is $v_z$. As before, we assume $0 < c_z < \bar{p}_z < v_z$ with $\bar{p}_z > \frac{v_z + c_z}{2}$.

The ensuing game between $P$ and $Z$ is summarized as follows (to be repeated in $t_6$ and $t_7$):

<table>
<thead>
<tr>
<th>$P/Z$</th>
<th>Keep$Z$</th>
<th>Deviate$Z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{p}_z - c_z$, $v_z - \bar{p}_z$</td>
<td>$\bar{p}_z - v_z$, $v_z - c_z$</td>
<td></td>
</tr>
<tr>
<td>$v_z - \bar{p}_z$, $v_z - v_z$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$-(I + I_z) + \frac{1}{1+r} \sum_{k=0}^{\infty} \left[ p^k(1-p)^{t-k}q_{c,0}u^kd^{t-k}B_P(Keep^C, Keep^C) \right] +$

$+ \sum_{t=2}^{3} \frac{1}{(1+r)^t} \sum_{k=0}^{t-1} \frac{t!}{k!(t-k)!} p^k(1-p)^{t-k}(q_{z,1}u_z^kd_z^{t-k}B_P(Keep^Z, Keep^Z) - M_P) ] >$

$-I + \frac{1}{1+r} \sum_{t=0}^{\infty} \left[ \frac{t!}{k!(t-k)!} p^k(1-p)^{t-k}q_{c,0}u^kd^{t-k}B_P(Keep^C, Keep^C) \right]$

This shows the trade off between respecting an original contract and using the strategic flexibility of options. While a contract reduces exposure to risk it also reduces the possibility of re contracting with a new client. A $M_P$ defined as in case A ensures that $P$ will be able to enjoy the benefits of switching to $Z$ while $P$ gets compensated for the period of break-up obtaining the equivalent to the highest possible benefit.

c) Case C: Further Flexibility

$P$ can further try to size the largest possible share of the excess benefits in the negotiation with $Z$. This involves solving the following problem:

$$\max_{\bar{p}_z \in \{ \bar{p}_z \}} \text{NPV}_P(\text{breakup})$$

s.t. $\text{NPV}_Z(\text{Keep}^Z, \text{Keep}^Z) > 0$

where

$$\text{NPV}_P(\text{breakup}) = -(I + I_z) + \frac{1}{1+r} \sum_{k=0}^{\infty} \left[ p^k(1-p)^{t-k}q_{c,0}u^kd^{t-k}(\bar{p}_z - c_z) \right] +$$

$$+ \sum_{t=2}^{3} \frac{1}{(1+r)^t} \sum_{k=0}^{t-1} \frac{t!}{k!(t-k)!} p^k(1-p)^{t-k}(q_{z,1}u_z^kd_z^{t-k}(\bar{p}_z - c_z) - M_P) ] >$$

and

$$\text{NPV}_Z(\text{Keep}^Z, \text{Keep}^Z) = \sum_{t=2}^{3} \frac{1}{(1+r)^t} \sum_{k=0}^{t-1} \frac{t!}{k!(t-k)!} p^k(1-p)^{t-k}q_{z,1}u_z^kd_z^{t-k}(v_z - \bar{p}_z)$$

The linearity of the problem allows to reduce it to find $\bar{p}_z$ such that $\text{NPV}_Z(\text{Keep}^Z, \text{Keep}^Z) = 0$. That is, $\bar{p}_z = v_z$.

Another possibility is for $P$ instead of selling the product to only one client, to sell a fraction to each of them. That is, at $t_6$ provide a proportion $f_z$ of the production to $C$ and $f_z$ to $Z$ (i.e. $t_6 + f_z = 1$). Of course, $C$ and $P$ will face a potential excess demand of their production, which in turn may impact on larger values of $v_z$ and $v_z$ respectively. The contract specifies only the

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7 We intend an expression $x \sim y$ to mean that $x \sim y$ but $|x - y|$ close to 0.

8 Here $\text{(Keep}^C, \text{Keep}^C)$ represents the situation in which the original price $\bar{p}_z$ is kept between $P$ and $C$ while $\text{(Keep}^Z, \text{Keep}^Z)$ reflects the agreement between $P$ and $Z$ on $\bar{p}_z$. 

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The comparison between the advantages of the RO approach and the optimal values can be assessed assuming the full rationality of the involved parties and the common knowledge of all the relevant future events. We illustrated this comparison in the light of a model of a client-provider problem. We saw that the incentives to keep or deviate from the contracts with \( C \) and \( Z \) are the same as before. Thus, the goal of \( P \) would be now:

\[
\max_{f_c \in (0,1)} NPV_P(f_c)
\]

s.t. \( NPV_C(f_c; \text{Keep}_C) > 0 \) and \( NPV_Z(1 - f_c; \text{Keep}_Z) > 0 \)

where

\[
NPV_P(f_c) = -(I + I_z) + \frac{1}{(1 + r)} \sum_{k=0}^{3} [p_c^k(1 - p)^{t-k}q_c,0u^k \hat{d}^{t-k}(\hat{p}_c - c_o)] + \sum_{t=2}^{3} \frac{1}{(1 + r)^t} \sum_{k=0}^{t} \frac{t!}{k!(t-k)!} p_c^k(1 - p)^{t-k}q_c,0u^k \hat{d}^{t-k}(f_c(\hat{p}_c - c_o) + q_z,1u^k \hat{d}^{t-k}(1 - f_c)(\hat{p}_z - c_z)]
\]

while

\[
NPV_C(f_c; \text{Keep}_Z) = \frac{1}{(1 + r)} \sum_{k=0}^{3} [p_c^k(1 - p)^{t-k}q_c,0u^k \hat{d}^{t-k}(v_c - \hat{p}_c)] + \sum_{t=2}^{3} \frac{1}{(1 + r)^t} \sum_{k=0}^{t} \frac{t!}{k!(t-k)!} p_c^k(1 - p)^{t-k}q_z,1u^k \hat{d}^{t-k}(f_c(v_c - \hat{p}_c)]
\]

and

\[
NPV_Z(\text{Keep}_Z, \text{Keep}_Z) = \sum_{t=2}^{3} \frac{1}{(1 + r)^{t-1}} \sum_{k=0}^{t} \frac{t!}{k!(t-k)!} p_c^k(1 - p)^{t-k}q_z,1u^k \hat{d}^{t-k}(1 - f_c)(v_z - \hat{p}_z)]
\]

Again, the linearity of the problem reduces it to the comparison between \( p_c - c_o \) and \( \hat{p}_z - c_z \). That is, the optimal level \( f_c^* \) is:

\[
f_c^* \begin{cases} 
1, & \text{if } (\hat{p}_c - c_o) > (\hat{p}_z - c_z) \\
0, & \text{if } (\hat{p}_c - c_o) < (\hat{p}_z - c_z) \\
\frac{1}{2}, & \text{otherwise}.
\end{cases}
\]

IV. Conclusion

We have examined the pros and cons of using a RO approach to contracts. We compare it to the rigidity predicated by the Neo-Institutional line of thought that sees flexibility as a source of additional transaction costs. We illustrated this comparison in the light of a model of a client-provider problem. We saw that adequate penalties enforce the relation if no outside parties exist, but allow the break-up of the relation to seek better opportunities. This possibility of switching partners can be fully captured in a real options framework and the optimal values can be assessed through game-theoretical analyses.

These formal explorations have been carried out assuming the full rationality of the involved parties and common knowledge of all the relevant future events. We think that the advantages of the RO approach still stand if we drop these assumptions and change towards a behavioral set of hypothesis (`a la [9]), in which the agents use heuristics instead of seeking optimal solutions. Further work involves exploring this intuition.

References


