Future Volatility Forecasting Models: An Analysis of the Brazilian Stock Market

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Abstract

Future volatility forecasting intrigues many scholars, researchers, and people from the financial markets. The model and methodology used for forecasting are fundamental for asset pricing in general, since future volatility deeply influences the final result. Thus, this study uses databases from the companies Vale and Petrobrás, in the period from July 1994 to August 2013, to test the Univariate, Bivariate, GARCH, and EGARCH models (also analyzing the results for the linear and quadratic methods) in order to assess the best model for forecasting future volatility. The results indicate that the quadratic method can better forecast future volatility than the linear method. The Univariate model showed the best results, proving that it is more efficient to use only short-term volatility for future volatility forecasting. If it were necessary to include long-term volatility, the Bivariate model would be the best, despite the GARCH and EGARCH models showing similar results.

Index terms— brazilian market; volatility forecasting; future volatility; historic volatility; average historic volatility.

1 Introduction

hen investors seek to invest their funds, the majority use profitability as the main criteria in making their decisions. However, it is also necessary to analyze risk and return, and volatility is one of the key measurement variables needed to make a good investment decision.

The standard, and simplest, way of measuring the volatility of an asset is by estimating the standard deviation of its returns. This measurement is usually defined as historic volatility. However, what is important for the financial market is not the historic value of the variance, but rather the value that is expected to prevail in the future. contrast, a high volatility asset presents abrupt oscillation, and is thus considered a high risk.

Therefore, the higher the volatility, the riskier the investment will be. When an asset has a low volatility it has a lower risk since its value changes slowly. In In this study, volatility is calculated as both the dispersion of asset returns in the stock market (standard historic volatility) and the variation between the highest and lowest price of an asset on a given day (average historic volatility).

There are several types of volatility mentioned in the literature on this subject, such as historic, future, expected, and implied volatility, which support investor analyses. However, future volatility is the type that matters the most, since it best describes the price dispersion of the underlying asset.

Due to the ease of obtaining historic and implied volatilities, these are often used in calculating the theoretical price of assets, even if this is not the best estimation method.

This study focuses on the Brazilian stock market. In the last five years, there has been growth in the average daily trading volume at an average annual rate of 7.0%(Compound Annual Growth Rate), with an emphasis on the options market, which grew at an average annual rate of 11.7% (Figure 1). Moreover, there is great potential in the Brazilian market because of the following reasons: the need for new listings by Brazilian companies wanting to obtain more capital for their investments; the growth of the middle class; and the wider dissemination and awareness of financial information due to the efforts of BM&FBOVESPA that launched a campaign in September 2010 with the strategy of increasing its total number of investors to 5 million within five years.

To give more consistency to the data and results, two highly liquid assets with a long trading history were chosen, namely, Petrobras PN (PETR4) and Vale PNA (VALE5). These are the two most negotiated shares on the Brazilian stock market in recent years. Their calls and puts are also the most negotiated options on BM&FBOVESPA, exchange where most trading in stock options is concentrated. Other stock options have low volume.

Figure 2 represents the daily returns of Petrobras and of Vale from July 5, 1994 to August 27, 2013, respectively.

2 Theoretical Framework

There are several definitions and concepts of volatility in the literature. According to Shiryaev (1999), there is no financial concept as discussed and as freely interpreted as volatility.

Usually this term is used in finance to denote the standard deviation of an asset's return. Thus, one of the variables for calculating volatility is the rate of return of an asset (ui) during a certain time interval i:

, where i = 1, 2, 3?, n. (

Where is the asset price in time i and is the asset price in time i -1.

Considering n + 1 observations, it is possible to calculate the asset's average return (?):

(2) The usual estimation that represents the variance of , is given by:

(3) Thus, volatility can be defined as .

3 a) Univariate and Bivariate Models of Historic Volatility

According to Katz and Cornick (2005), historic volatility is generally used to calculate option price. However, option value is not defined by historic volatility, but by future volatility.

Because of this, some experiments were conducted in order to calculate future volatility based on historic volatility. Two of these models are used in this study: univariate and multivariate models of historic volatility.

In the univariate model of historic volatility, two measures are used to calculate volatility: standard historic volatility based on the standard deviation of the logarithmic returns, as can be seen in equation (3); and average historic volatility, as can be seen in equation (**??**), according to Katz and Cornick (2005):(4)

Where m represents the period selected, is the maximum of the asset in the period and is the minimum of the asset in a certain period.

On the other hand, the bivariate model of historic volatility uses a short-term historic volatility measurement and adds a long-term historic volatility measurement for future volatility forecasting.

Caspary (2011) uses both univariate and bivariate regressions to obtain a relationship between standard historic volatility and future volatility. In his study, he analyzed the twenty-seven most liquid shares of Bovespa, in addition to IBOVESPA, to forecast future volatility. In both models, the results were satisfactory and mean reversion was observed, i.e., lower values of Bollerslev (1986) expanded Engle's model in order to allow the conditional variance to be modeled as an autoregressive-moving-average process (ARMA). According to Gujarati (2005), the generalized autoregressive conditional heteroskedasticity model (GARCH), as the name implies, is a generalization of the ARCH model, where the conditional variance at a certain time depends on disturbances and past conditional variances.

GARCH (1,1) is the simplest and most used GARCH model. GARCH (1,1) was used in this study because it is the GARCH model series that best fits, and because there is autocorrelation between the residues found in AR(1) regression, whereas there is no autocorrelation between residues found in AR(2) regression. Equation 5 follows, and represents this model: , and ? the weight related to . The sum of these weights is 1, like the equation: (6) Besides being a more advanced model than ARCH, it can also be considered an extension of the exponentially weighted moving average model (EWMA), since the long-term variance rate is taken into account, which influences the calculation of today's variance.

Therefore, the GARCH model was used in this study and its results will be presented in the results section.

The exponential generalized autoregressive conditional heteroskedasticity model (EGARCH) will also be used because it is an even more developed model than GARCH, since it uses asymmetry, taking into consideration that each rise and fall in the asset's value is weighted differently in volatility.

This model created by Nelson (1991) aimed to develop a multivariate version of exponential ARCH and a satisfactory asymptotic theory for estimating parameters of maximum likelihood. (7) Where g(Zt) = ? + ? (| Zt | ? E (| |)),

is the conditional variance, ?, ?, ?, ? and ? are coefficients, and can be a standard normal variable or come from generalized error distribution. As well as the GARCH model, EGARCH (1,1) will be used in this study, rather than the other EGARCH series, since it presents autocorrelation between the regression residuals, adjusting better to the model.

When Morais and Portugal (1999) analyze which model best predicts the volatility of IBOVESPA in stable or troubled periods, they conclude that the GARCH model (deterministic model) presents superior results in a certain period of calm in the market, while the stochastic model obtains more satisfactory results in periods of crisis. Wang (2007), when analyzing historic models, moving average volatility, GARCH and EGARCH volatility, as well as implied volatility in order to forecast the future volatility of shares, government bonds and foreign exchange market, concludes that the implied volatility model had the best results in forecasting future volatility, although all models had low influence. ??iglewski (2004), on the other hand, when comparing several models for forecasting future volatility, concludes that, in general, the historic volatility model provides better results for predicting short-term and long-term future volatility. According to ??iglewski (2004), the GARCH model requires a larger sample size to give a better estimate. Therefore, when daily data are used in the model, GARCH achieves satisfactory results for forecasting volatility in a horizon of less than three months.

4 III.

5 Methodology and Database

The database used in the study is comprised of a history of share prices used in determining the historic volatility of the underlying asset for the different time horizons and consequent establishment of volatility according to predefined periods. Economática was the system selected for the survey of the asset price database. Besides allowing the acquisition of information on various shares for long periods of time, it also enables the extraction of a history of share prices that is already adjusted to the payment of dividends. Thus, the calculation of variations in asset prices can be made directly, without additional adjustments, since it is possible to obtain them already adjusted. The representation of the EGARCH(p,q) model follows in equation 7:

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For the current study, Petrobras (PETR4) and Vale (VALE5) shares were selected for being high liquidity shares, and presenting high daily trading volume with a long history, which provides a better analysis. The series obtained consists of daily closing, opening, and, maximum and minimum values of the assets from July 5, 1994 to August 27, 2013. The selected period includes a good historic record for analysis because of the Real Plan (Plano Real -a Brazilian Economic Program) and, to an extent, controlled inflation, which allows for the acquisition of satisfactory results. It is noteworthy that both, periods of crisis and periods of economic expansion, were used, and all trading days were considered.

7 a) Univariate and Bivariate Models of Historic Volatility

For the application of these two models, the methodology suggested by Katz and Cornick (2005) was used.

First, a stock is selected. In this study, the Petrobras (PETR4) and Vale (VALE5) shares were selected. Then, the period of analysis, from July 5, 1994 to August 27, 2013 was selected.

Using each reference date selected, historic volatility was calculated for 30 days prior (m1 = 30) and future volatility was calculated for 10 days (n1 = 10) immediately after the reference date. Figure 3 From this point on, another reference date is selected, and historic and future volatility are calculated as shown in the model. This process is repeated until the last reference date is selected.

In the univariate model, two measures of historic volatility are analyzed, namely, standard historic volatility and average historic volatility. With this, two series are used: one representing the relationship between standard historic volatility and standard future volatility, and the other representing the relationship between average historic volatility and standard future volatility.

This second study was done to assess if the multivariate model provides better estimates of future volatility than the model with a single variable.

In this study, two measures of historic volatility -a short-term one and a long-term one-were used to predict future volatility.

For each selected data on a certain date, m1 data preceding the reference data were selected, and historic volatility was short-term. After the last data of m1, m2 data were selected for calculating long-term historic volatility. In contrast, future volatility was calculated based on the m3 data selected after the reference data.

In this case, m1 was equal to 30, m2 was equal to 70, and m3 equal to 10. After calculating these three volatilities, the next reference data was selected, the volatilities were calculated, and so on.

8 b) GARCH and EGARCH Models

To work with the GARCH and EGARCH models, EVIEWS was used to obtain long-term volatility.

Thus, from the returns of the Petrobras (PETR4) and Vale (VALE5) shares, the equation was estimated using the GARCH method. The daily variance for every share and therefore the annualized volatility for each of the periods were obtained as a result.

The same procedure was carried through for the EGARCH method, with long-term volatility as the final result.

IV.

9 Results

Having obtained the historic series of the shares and applied the methodology presented above, historic and future volatilities were obtained. With these results, linear and quadratic regressions were applied to better analyze the results, verifying the reliability and comparing the various models used in the study.

As previously mentioned, some models were used to calculate the future volatility of the preferred shares of Petrobras and Vale: the univariate model of historic volatility, the bivariate model of historic volatility, GARCH and EGARCH. In this section, the results of each model will be described.

10 a) Univariate Model of Historic Volatility

First, the daily closing prices and daily minimum and maximum share prices of Petrobras and Vale from July 5, 1994 to June 30, 2011 were collected for the calculation of share returns.

Then, standard historic volatilities, average historic volatilities, and standard future volatilities were calculated.

Two series related to the Petrobras share are shown in the graph in figure 5: the first illustrating the relationship between standard historic volatility and standard future volatility, and the second illustrating the relationship between average historic volatility and The dotted line represents the quadratic regression obtained from the relationship between standard historic volatility and standard future volatility and the solid line represents the quadratic regression obtained from the average historic volatility and standard future volatility. Tables 1 and 2 show the results of the regressions carried out using the SPSS statistical analysis tool. From the graph in Figure 5, it is clear that the volatilities exhibit similar behavior for about 40% of each calculation after which there is a slight deviation from the average historic volatility. It can be noticed that the volatility displays mean reversion, i.e., low levels of historic volatility lead to higher levels of future volatility, while high levels of historic volatility imply lower levels of future volatility.

For almost all volatilities, the calculation using average volatility provides a better estimate of future volatility than the standard measurement. This occurs because there is a greater reliability upon the average as a measure of volatility, and its volatility better explains future volatility, since it has a slightly higher R^2, as detailed in the results obtained using regression.

Two series related to the Vale share were also used. Both series are shown in the chart in figure 6: the first illustrates the relationship between standard historic volatility and standard future volatility, and the second illustrates the relationship between average historic volatility and standard future volatility. The x-axis (horizontal) represents historic volatility and the y-axis (vertical) future volatility. As observed for Petrobras, from the graph with the Vale volatilities we observe that the volatilities exhibit a similar behavior up to approximately 45% of each calculation.. Additionally, the volatility displays mean reversion, i.e., low levels of historic volatility causing higher levels of future volatility, while high levels of historic volatility imply lower levels of future volatility, showing greater influence than in the Petrobras shares.

By having a slightly higher R^2 in the average historic volatility model, it can be seen that its volatility better forecasts future volatility, being a more reliable measure.

For the models shown above, taking into account the R^2 presented in this section, the Petrobras share provides better results than the Vale share when calculating future volatility.

11 b) Bivariate Model of Historic Volatility

In the multivariate model, in addition to shortterm historic volatility, a long-term historic volatility was added, differing from the univariate model presented earlier.

As this model presents a two variable function, the result would be a three-dimensional graph. For ease of viewing, some short-term volatilities were established and a certain future volatility was obtained according to long-term volatility.

Both linear and quadratic methods were used for calculating future volatility, according to the study suggested above. The two models presented satisfactory results for both the Petrobras and Vale shares.

Using linear regression for the Petrobras shares, the results were: the higher the short-or long-term volatility, the higher the future volatility.

Table **??** shows the result of the linear regression of short-and long-term historic volatility for calculating future volatility and figure 7 shows it as a graph.

12 Table 4 : Linear Regression between Long-Term Historic

Volatility and Short-Term Historic Volatility for the Petrobras Shares using the Bivariate Model. As it is linear, if the short-term volatility is the same, an increase in long-term volatility generates an increase in future volatility.

As with the linear method, the same methodology was followed for calculating the results of the Petrobras shares using the quadratic method. The results can be seen in table **??** and figure 8. Using the quadratic method, it can be seen that the higher the short-term volatility, the higher is the future volatility, presenting a proportional relationship. At a short-term historic volatility of 50% or less, any variation in historic volatility implies a larger variation in future volatility. After 50%, the variation in this volatility has less and less impact on the variation of future volatility. It can be observed that future volatility is very similar when short-term historic volatility is 125% or 150%.

Including long-term volatility in the analysis, it can be observed that at long-term volatility of approximately 70%, the higher the long-term volatility, the higher the future volatility. From then on, the increase in long-term volatility generates minor impact on future volatility, proving there is mean reversion. Considering a short-term volatility at 75%, when longterm volatility is at 50% or at 90%, future volatility is at 65%.

Thus, for the Petrobras PN shares, one instance of high volatility will barely remain in this baseline for a long period of time. Table **??** aids in understanding the aforementioned with regard to the analysis of volatility. Table **??** : Linear regression between long-term historic volatility and short-term historic volatility for the Vale shares using the bivariate model.

Using linear regression to obtain the future volatility model for the Vale shares, the same results are found as in the application of linear regression for the Petrobras share, i.e., the higher the short-and long-term volatility, the higher the future volatility. In order to check the results for the quadratic model, a regression between short-term historic volatility, short-term squared volatility, long-term volatility, and long-term squared volatility was performed to calculate future volatility. Table **??** and figure 10 show the results.

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Table **??** : Quadratic regression between long-term historic volatility and short-term historic volatility for the Vale shares using the bivariate model.

It can be observed in figure 10 that, in applying the quadratic method of regression for the Vale shares, it becomes clear that at short-term historic volatility of 50% or less, any variation in short-term historic volatility generates greater variation in future volatility. After 50%, the volatility of this variation will have less and less impact on the variation of future volatility, until 125%, when an increase above this value in short-term historic volatility leads to lower future volatility. By including long-term volatility in the analysis, it is clear that an increase in this volatility causes less and less variation in future volatility, with a tendency to remain constant during periods of high volatility. For instance, considering a short-term volatility at 50%, when there is a long-term volatility at 20%, the result is a future volatility of 41%. If long-term volatility is at 30%, future volatility is at 44%. By increasing long-term volatility to 40%, future volatility grows to 46%. Table **??** sums this up.

Table **??** : Relationship between short-and long-term historic volatilities and future volatility for the Vale shares from quadratic regression using the bivariate model of historic volatility.

As R^2 is higher with the quadratic model for both Petrobras and Vale, it can be seen that short-and long-term volatility that best determines future volatility is obtained by using this model, which presents more reliable results.

For the models shown above, the Petrobras share provides better results than the Vale share when calculating future volatility.

14 c) GARCH Model

For application of the GARCH model, EViews statistical package was used to calculate long-term historic volatility. From the data obtained with the bivariate model of historic volatility, the long-term historic volatility calculated for this model was replaced by the volatility calculated using EViews.

Linear and quadratic regressions were performed for both the Petrobras and Vale shares with the volatility results obtained.

Applying the linear regression method for the Petrobras shares, it is clear that there is a relationship between the dependent and independent variables, since they have high student's t-distribution, according to the results obtained in Table 10.

Table10 : Linear regression between long-term historic volatility and short-term historic volatility for the Petrobras shares using the GARCH model.

For a better understanding of the results, shortterm volatility was fixed and future volatility was calculated using the variation of long-term volatility. Using the equation obtained from the regression, it can be seen that the higher the long-term volatility, the lower is the future volatility, and the higher the short-term volatility, the higher is the future volatility. The graph from figure 11 shows this relationship. This model presents different results than the bivariate model of historic volatility. Considering a shortterm historic volatility at 50% and a long-term historic volatility at 30%, a future volatility of 45% is obtained using the GARCH model. Applying the same historic volatilities in the historic bivariate model, a future volatility of 43% is obtained.

When using a short-term historic volatility at 50% and a long-term historic volatility at 80%, a future volatility of 44% is obtained using the GARCH model. Applying the same historic volatilities in the historic bivariate model, a future volatility of 48% is obtained. That is, using the GARCH model, the higher the longterm historic volatility, the lower the future volatility. On the other hand, using the bivariate model, it can be seen that the higher the long-term historic volatility, the higher is the future volatility. Figure 12 Applying quadratic regression for the results of the Petrobras share, it is concluded from student's t that linear and squared long-term volatility and linear and squared short-term volatilities influence future volatility.

Table 11 : Quadratic regression between long-term historic volatility and short-term historic volatility for the Petrobras shares using the GARCH Model.

For short-term volatility, the results were similar to those obtained from the bivariate model, i.e., the higher the short-term volatility, the higher the future volatility.

However, when analyzing long-term volatility, it is clear that, in the GARCH model, the higher the volatility, the lower the future volatility. On the other hand, the bivariate model displays mean reversion. A linear regression between the short-and longterm historic volatility was also performed to calculate the vale share's future volatility. The results were similar to those obtained for the Petrobras share, i.e., the higher the long-term volatility, the lower the future volatility. The result of this regression is in Table 12. Table 13 : Quadratic regression between long-term historic volatility and short-term historic volatility for the Vale shares using the GARCH Model.

All variables explain future volatility, since they show student's t-distribution, according to the regression result in table 13.

As can be observed in figure 15, until short-term volatility reaches 125%, the increase in this volatility generates an increase in future volatility. From then on, it can be seen that an increase in short-term volatility generates a decrease in future volatility.

However, the increase in long-term volatility generates an increase in future volatility to 50% when the curve is reversed, and an increase in long-term volatility above 50% influences a decrease in future volatility, with mean reversion.

In fixing short-term volatility at 50% and using a long-term volatility of 40%, a future volatility of 47% is obtained. By applying the same short-term volatility and changing the long-term volatility to 90%, the result is a future volatility of 44%. In accordance with the results obtained in Table 14, it can be observed that there is a relationship between the dependent and independent variables, since they have a high student's t-distribution.

Table 14 : Linear regression between long-term historic volatility and short-term historic volatility for the Petrobras shares using the EGARCH model.

Short-term volatility was fixed and future volatility was calculated from the variation of long-term volatility. Through the equation obtained from the regression, the higher the long-term volatility, the lower the future volatility, and the higher the short-term volatility, the higher the future volatility.

However, the higher the long-term volatility, the lower the future volatility, since the coefficient of this variable in the EGARCH model is more negative than in the GARCH model. That is, a 10% increase in long-term volatility generates a reduction of 0.32% using the GARCH model, while this reduction is 0.49% using EGARCH.

Figure 16 : Relationship between short-and long-term historic volatilities and future volatility for the Petrobras share from linear regression using the EGARCH model.

Using quadratic regression for the results of the Petrobras share, it is concluded that long-term volatility and squared long-term volatility do not influence future volatility. On the other hand, linear and squared shortterm volatilities influence future volatility (table 15).

Table 15 : Quadratic regression between long-term historic volatility and short-term historic volatility for the Petrobras shares using the EGARCH model.

As observed in the chart in figure 17, the higher the short-term volatility, the higher the future volatility. As mentioned above, long-term volatility does not explain future volatility, as it does not provide meaningful results.

A linear regression between short-and longterm historic volatility for calculating the vale share's future volatility was also used, as can be seen in table 16.

Table 16 : Linear regression between long-term historic volatility and short-term historic volatility for the Vale shares using the EGARCH model.

Applying linear regression to obtain the future volatility model for the Vale shares, it was found that the results were similar to those from the GARCH model, i.e., according to the equation obtained using the regression, the higher the long-term volatility, the lower the future volatility.

However, using EGARCH, the higher the longterm volatility, the lower the negative influence in future volatility, since the coefficient of this variable in this model is bigger than the coefficient of this same variable using the GARCH model. That is, a 10% increase in long-term volatility generates a reduction of 0.61% using the GARCH model, while this reduction is 0.36% using EGARCH. Similar to what happened with the application of the GARCH model, the increase in short-term volatility generates an increase in future volatility up to 125%. From then onwards, it can be seen that the increase in short-term volatility generates a decrease in future volatility. In the chart above, the future volatilities resulting from short-term volatility of 150% are smaller to the future volatilities obtained through short-term volatility of 125%. The increase in long-term volatility provides an increase in future volatility to 50% when the curve is reversed, and the increase in long-term volatility above 50% influences a decrease in future volatility, proving the mean reversion. Therefore, these results from the EGARCH model are similar to those of the GARCH model. However, it is clear that, with long-term future volatility at 80%, any increase in this volatility generates a negative variation in future volatility greater than in the GARCH model.

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When using short-term volatility at 50% and a 10% future volatility, the EGARCH model's future volatility is 42%, while that of the GARCH model is 45%. Applying a 70% long-term volatility, the EGARCH model's future volatility is 46% while the GARCH model is 47%. In other words, the variation is small in both models. However, if the long-term volatility is 110%, the EGARCH model's future volatility is 32%, while the GARCH model's is 40%. Similar to the GARCH model, the Petrobras and Vale quadratic models better explain future volatility than the linear models. Moreover, the Petrobras share provides better results for calculating future volatility than the Vale share.

V.

16 Conclusion

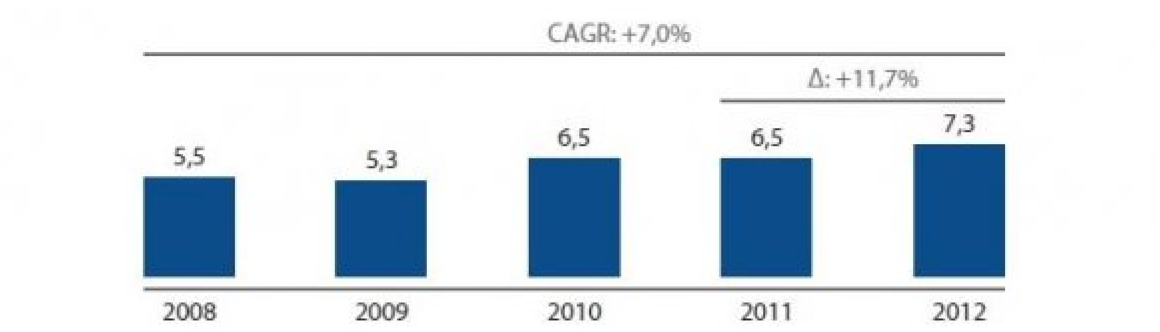
This study sought to test the effectiveness of certain models in forecasting future volatility since volatility is one of the most difficult variables to calculate, in addition to having a significant impact on option price and on the estimation of future share value.

The results presented indicate that in all applied models, it is possible to better predict future volatility using the quadratic method rather than the linear method, since volatility models tend to be nonlinear and the R^2 from the regressions was higher when using the quadratic method.

It can also be concluded that the univariate model presents better results than the bivariate model. Additionally, the inclusion of another variable worsened [1]

1

Figure 1: Figure 1 :



						CAGR	Variation
Markets	2008	2009	2010	2011	2012	2008-2012	2012/2011
Spot	5.162,30	4.943,70	6.031,60	6.096,30	6.861,30	7,40%	12,50%
Forward	177,80	96,50	147,40	118,00	103,40	-12,70%	-12,40%
Options	180,20	245,00	307,90	276,30	280,10	11,70%	1,40%
Total	5.525,50	5.286,80	6.488,60	6.491,60	7.250,70	7,00%	11,70%

Figure 2: W

2 $u_i = \ln(S_i / S_{i-1})$

Figure 3: Figure 2 :

S_i

Figure 4:

5 $$\bar{u} = \frac{1}{m}\sum_{i=1}^{m} u_i$$

Figure 5: (5)

$$\sigma_n^2 = \frac{1}{m-1}\sum_{i=1}^{m}(u_i - \bar{u})^2$$

Figure 6: Volume

σ_n

Figure 7:

3 $$v = 0{,}627\left(\frac{1}{m}\right)\sum_{i=0}^{m-1} ln(H_{k-1}/L_{k-1})$$

Figure 8: Figure 3 :

4 H_{k-1}

Figure 9: Figure 4 :

I_{k-1}

Figure 10: C

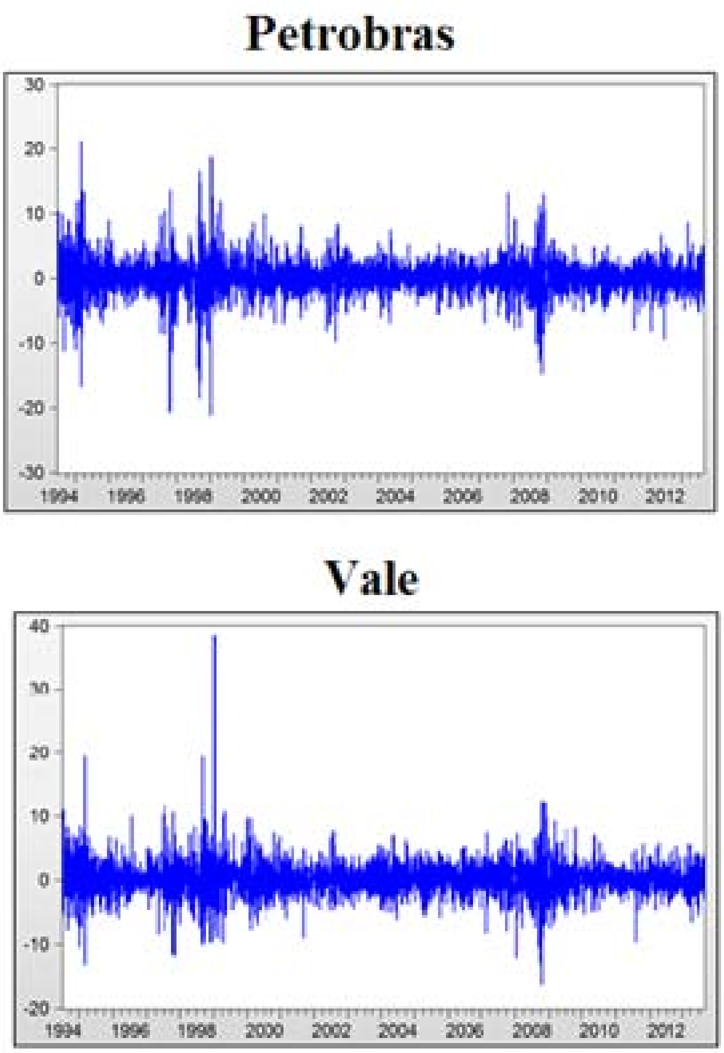
5 

Figure 11: Figure 5 :

6 $\sigma_n^2 = \gamma V_L + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2$

Figure 12: Figure 6 :

7 u_{n-1}^2

Figure 13: Figure 7 :

5 σ_{n-1}^2

Figure 14: Table 5 :

8 u_{n-1}^2

Figure 15: Figure 8 :

6 σ_{n-1}^2

Figure 16: Table 6 :

9 $\gamma + \alpha + \beta = 1$

Figure 17: Figure 9 :

$$Log\,\sigma_t^2 = \omega + \sum_{k=1}^{q} \beta_k g(Z_{t-k}) + \sum_{k=1}^{p} \alpha_k \log(\sigma_{t-k}^2)$$

Figure 18: VolumeFuture

10 Z_t

Figure 19: Figure 10 :

11 Z_t

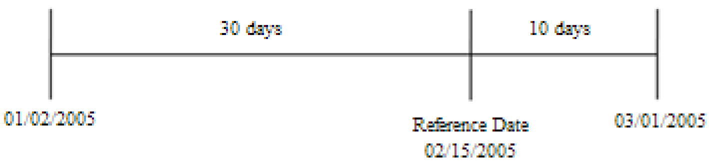
Figure 20: Figure 11 :

σ_t^2

Figure 21:

12 Z_t

Figure 22: Figure 12 :



13

Figure 23: Figure 13 :

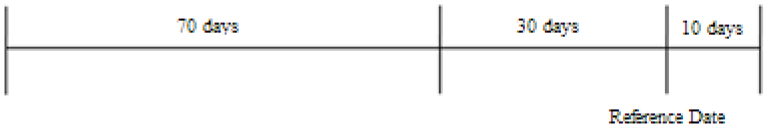


Figure 24:

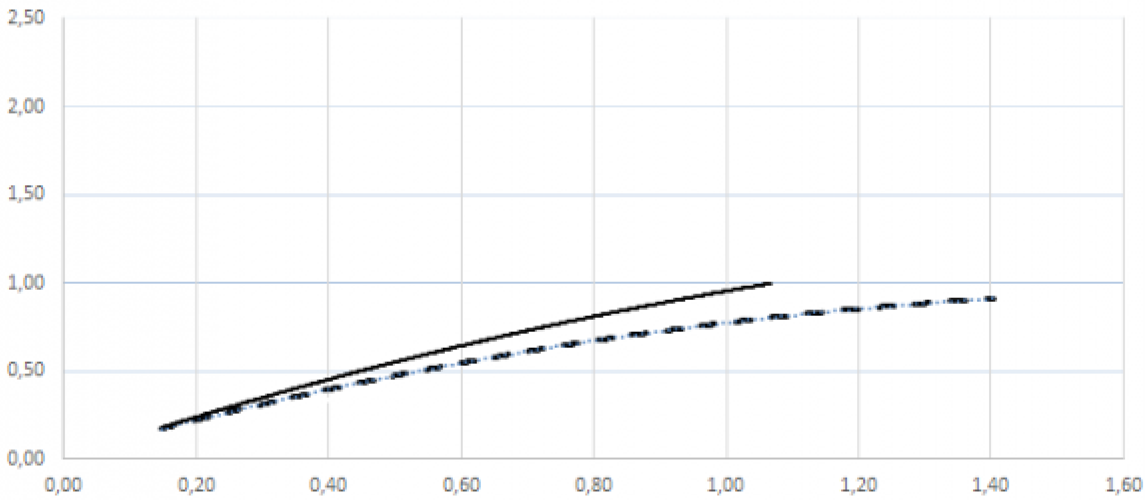


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Figure 25: Figure 14 :

	Unstandardized Coefficients		Standardized Coefficients	t	Sig.
	B	Std. Erro	Beta		
(Constant)	0,026	0,012		2,090	0,037
Std. Historic Volatility	1,041	0,049	0,921	21,345	0,000
Std. Historic Volatility ^{2}	-0,0293	0,038	-0,336	-7,788	0,000

	Sum of Squares	df	Mean Square	F	Sig.
Regression	86,298	2	43,149	1348,200	0,000
Residual	150,200	4693	0,032		
Total	236,498	4695			

R	R^2	Ajusted R^2	Std. Error
0,6041	0,3649	0,3646	0,1789

15

Figure 26: Figure 15 :

16 CONCLUSION

	Unstandardized Coefficients		Standardized Coefficients	t	Sig.
	B	Std. Erro	Beta		
(Constant)	0,002	0,015		0,135	0,893
Avg. Historic Volatility	1,243	0,070	0,837	17,863	0,000
Avg. Historic Volatility ^{2}	-0,290	0,068	-0,200	-4,276	0,000

	Sum of Squares	df	Mean Square	F	Sig.
Regression	98,230	2	49,115	1667,017	0,000
Residual	138,268	4693	0,029		
Total	236,498	4695			

R	R^2	Ajusted R^2	Std. Error
0,6445	0,4154	0,4151	0,1716

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Figure 27: Figure 17 :

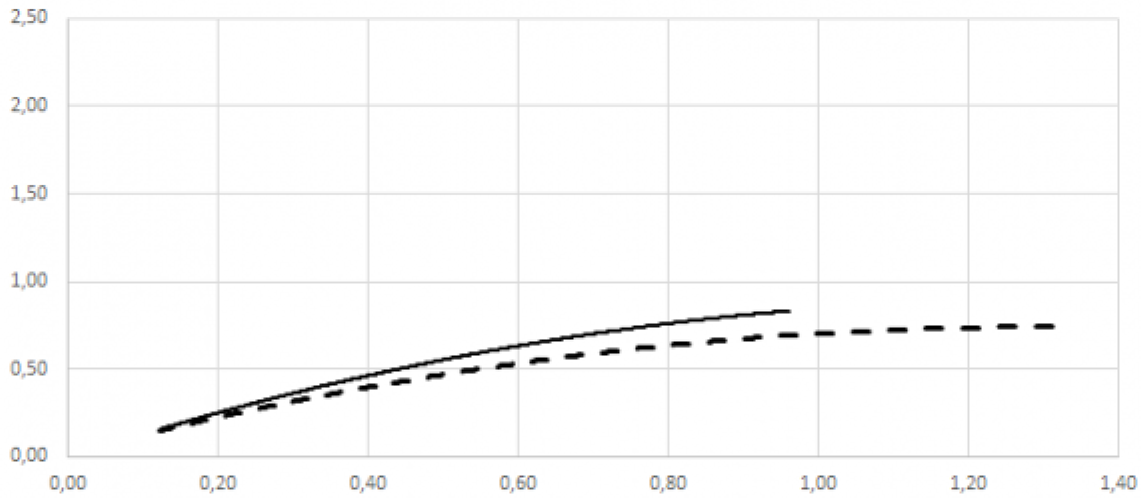

Figure 28: VolumeFuture

	Unstandardized Coefficients		Standardized Coefficients	t	Sig.
	B	Std. Erro	Beta		
(Constant)	0,018	0,012		1,445	0,148
Std. Historic Volatility	1,116	0,049	0,955	22,718	0,000
Std. Historic Volatility ²	-0,432	0,041	-0,448	-10,651	0,000

	Sum of Squares	df	Mean Square	F	Sig.
Regression	59,683	2	29,8416	980,170	0,000
Residual	142,971	4696	0,0304		
Total	202,655	4698			

R	R^2	Ajusted R^2	Std. Error
0,5427	0,2945	0,2942	0,1745

	Unstandardized Coefficients		Standardized Coefficients	t	Sig.
	B	Std. Erro	Beta		
(Constant)	-0,006	0,015		-0,405	0,686
Avg. Historic Volatility	1,381	0,077	0,872	17,914	0,000
Avg. Historic Volatility ²	-0,529	0,084	-0,306	-6,296	0,000

	Sum of Squares	df	Mean Square	F	Sig.
Regression	68,037	2	34,018	1186,699	0,000
Residual	134,618	4696	0,029		
Total	202,655	4698			

R	R^2	Ajusted R^2	Std. Error
0,5794	0,3357	0,3354	0,1693

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Figure 29: Figure 18 :

	Unstandardized Coefficients		Standardized Coefficients	t	Sig.
	B	Std. Erro	Beta		
(Constant)	0,096	0,007		14,446	0,000
L.T. Historic Volatility	0,091	0,018	0,075	5,176	0,000
S.T. Historic Volatility	0,622	0,016	0,551	37,975	0,000

	Sum of Squares	df	Mean Square	F	Sig.
Regression	82,970	2	41,485	1286,171	0,000
Residual	149,114	4623	0,032		
Total	232,084	4625			

R	R^2	Ajusted R^2	Std. Error
0,5979	0,3575	0,3572	0,1796

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Figure 30: Table 17 :

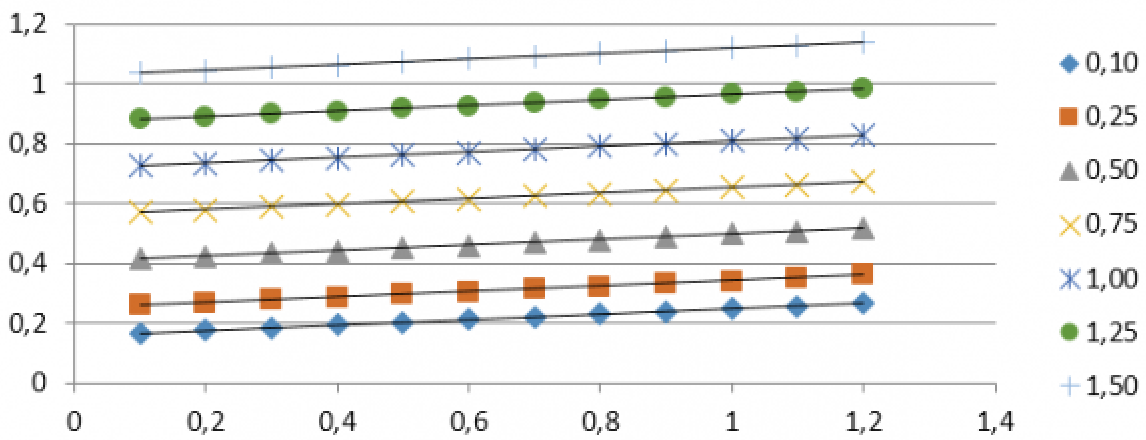

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Figure 31: Figure 19 :

	Unstandardized Coefficients		Standardized Coefficients	t	Sig.
	B	Std. Erro	Beta		
(Constant)	-0,027	0,019		-1,430	0,153
L.T. Historic Volatility	0,336	0,074	0,277	4,565	0,000
L.T. Historic Volatility ^{2}	-0,231	0,060	-0,223	-3,834	0,000
S.T. Historic Volatility	0,919	0,054	0,814	17,142	0,000
S.T. Historic Volatility ^{2}	-0,234	0,039	-0,270	-6,071	0,000

	Sum of Squares	df	Mean Square	F	Sig.
Regression	84,758	4	21,189	664,626	0,000
Residual	147,326	4621	0,032		
Total	232,084	4625			

R	R^2	Ajusted R^2	Std. Error
0,6043	0,3652	0,3647	0,1786

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Figure 32: Figure 20 :

1

Figure 33: Table 1 :

2

Figure 34: Table 2 :

3

Figure 35: Table 3 :

Future Volatility Forecasting Models: An Analysis of the Brazilian Stock Market the results, proving that it is more efficient to use only short-term volatility to forecast future volatility.

It was found that, in a similar way to what was reported by Caspary (2011), both the univariate and the bivariate models showed characteristics that lead to observing a mean reversion trend.

If there is a need to include long-term volatility, the bivariate model of historic volatility showed better results, despite the GARCH and EGARCH models producing very similar, although slightly lower, results. This contradicts the findings of Morais and Portugal (1999), who concluded that the GARCH model provided better results than the other models.

Taking R^2 into account, the model achieving the best results was the average historic volatility model using the univariate method of historic volatility. Therefore, it is the most suitable model for forecasting future volatility.

The results for the Petrobras share were better than the Vale share in all models, since Petrobras had a higher R^2 in all of them.

Future studies could improve results by using alternative models or even applying the models mentioned in this work on other shares with liquidity in the Brazilian market.

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