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Fat Tails, Value at Risk, and the Palladium Returns

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GJMBR-B Classification: *JEL Code: C46; C58; G10*



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Fat Tails, Value at Risk, and the Palladium Returns

Turen Guo [α], Jianhua Ding [σ] & Bin Guo [ρ]

Abstract- The past decade has witnessed the rapid growing of the world palladium market. Thus, it is even more important to develop effective quantitative tools for risk management of palladium assets at this moment. In this paper, we investigate five different types of widely-used statistical distributions and employ the industry standard risk measurement, Value at Risk, for risk management of daily palladium spot returns. We first apply four different criteria to compare the goodness of fit of the five distributions, and then calculate the VaRs based on the parameters estimated from the first step. Our results indicate the Skewed t distribution has the best in-sample fitting and generate VaR values closest to the nonparametric historical VaR values.

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I. Introduction

During the recent decades, many researchers and practitioners have been drawn by various kinds of alternative investments to diversify their portfolios. Market participants are extremely excited about the potential investment vehicles which could serve as safe hedging for conventional asset classes, such as stocks, foreign exchanges and fixed income bonds. The precious metal of palladium, which has been widely used in automotive, chemical, electrical, jewellery and dental industries, naturally serves as a popular candidate. Similar as any other products, the price of palladium are determined by the supply and demand. Currently, the two largest producers are Russia and South Africa, which consist of more than 75% of annual world mine supplies. On the demand side, the rising giant, China, accounts for more than 70% of annual global demand increases of palladium. Moreover, the increasing demand in China is mainly responsible for the price increase since the year of 2004. According to the United States Geological Survey, the global mine production of palladium was slightly more than 250 tons in 2010, and the world demand of palladium was nearly 350 tons in 2010.

Since so many investors have been drawing to the global palladium market and the market capitalization has increased so dramatically during the last decade, it becomes a more urgent task for market practitioners and academia to develop effective risk management tools. In this paper, we focus on the development of quantitative risk management tools. We take advantage of the widely-used concept, Value at Risk (VaR), and investigate empirical performance of a variety of heavy-tailed distributions in risk measures calculations. Our statistical distributions cover: the normal, the Student's *t*, the Skewed *t*, the normal inverse Gaussian (NIG), and the generalized hyperbolic (GH) distributions. Our results indicate the Skewed *t* distribution could generate most suitable VaR values and outperform other types of popular statistical distributions.

a) Literature Review

Since these five different statistical distributions were introduced into the literature, there have been extensive studies on their performance in fitting asset returns. Both the normal and the Student's *t* distributions have been in the literature for more than a hundred years (see Helmert, 1876). Hansen (1994) introduces the Skewed t distribution and discussed its performance in fitting the US stock returns. There are many other types of asymmetric *t* distribution, and we chose the one in Hansen (1994) for its simplicity. Also, Barndorff-Nielsen (1977) developed the GH distribution for the US stock returns. As surveyed by see Figueroa-Lopez, et al. (2011) that the NIG distribution is one of the most popular subclasses of the GH distribution in financial modeling, and thus we are also interested in it in this paper. We have also investigated some other subclasses of the GH distribution, such as the normal reciprocal inverse Gaussian (NRIG), and the results are similar and available upon request. Our work is closely related to the work in Guo (2017a). Guo compared these five widely-used statistical distributions in fitting the SP 500 index returns and showed the Skewed *t* distribution has the best in-sample fitting and predicts the most accurate risk measure values. One could also see other similar studies in Bueno, Fortes and Vlachoski (2017) and Kayaba, Hirano, Baba, Matsui and Ueda (2017).

In this paper, we reconsider these five types of statistical distributions but focus on the precious metal market. Our special interests are on palladium, one of the rare metals which have gained increasing attentions in the financial market and academia. There are quite a few studies on palladium asset returns. Adrangi and

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Chatrath (2002) focused on ARCH-type models and provided evidence of nonlinear dependencies in palladium and platinum futures markets with controls for seasonality and contract-maturity effects generally explaining the nonlinearities in the data. Diaz (2015) investigated the spots prices of the two scarce precious metals, platinum and palladium. Diaz found intermediate memory in the return structures of both precious metals, which implies the instability of platinum and palladium returns' persistency in the long run. Auer (2015) also used GARCH models to investigate the impact of the specific calendar day on the conditional means of palladium returns, and showed that during the period from July 1996 to August 2013 there is no significant impact of the specific calendar day observed. Some researchers also investigated the hedging effect of the palladium. For instance, Pierdzioch, Risse and Rohloff (2016) used Bayesian additive regression trees to reexamine whether investments in precious metals are a hedge against exchange-rate movements and showed that investments in gold and silver are strong hedges against depreciations of major exchange rates but the hedging properties of palladium and platinum are mainly confined to the Australian dollar and Canadian dollar. Similar studies could also be found in Hammoudeh, Malik and Mc Aleer (2011). Finally, Caporale, Spagnolo and Spagnolo (2017) adopted a vector autoregressive model to investigate the relationship between macro news and commodity returns and indirectly showed the palladium could serve as a safe asset to the stock market in US.

In this paper, our main interests are developing an effective quantitative risk management tool based on the concept of VaR. The paper is structured as follows. In Section 2, we introduce the heavy-tailed distributions. Section 3 summarizes the data. The estimation results are in Section 4. Finally, we conclude in Section 5.

II. Heavy-Tailed Distributions

In this section, we introduce four types of widely-used heavy-tailed distribution in addition to the normal distribution: (i) The Student's *t* distribution; (ii) The Skewed *t* distribution; (iii) The normal inverse Gaussian distribution (NIG); and (iv) The generalized hyperbolic distribution (GH). All the distributions have been standardized to ensure mean and standard deviation equal to zero and one respectively. Their probability density functions are given as follows.

(i) Student's t Distribution

$$f(e_t \mid \nu) = \frac{\Gamma(\frac{\nu+1}{2})}{\Gamma(\frac{\nu}{2})[(\nu-2)\pi]^{1/2}}\left(1+\frac{e_t^{\,2}}{(\nu-2)}\right)^{-\frac{\nu+1}{2}}, \quad (1)$$

Where ν indicates degrees of freedom and e_t is daily equity market index return.

(ii) Skewed t Distribution

$$f(e_t \mid \nu, \beta) = \begin{cases} bc\left(1+\frac{1}{\nu-2}\left(\frac{be_t+a}{1-\beta}\right)^2\right)^{-(\nu+1)/2} & e_t < -a/b \\ bc\left(1+\frac{1}{\nu-2}\left(\frac{be_t+a}{1+\beta}\right)^2\right)^{-(\nu+1)/2} & e_t \geq -a/b \end{cases}, \quad (2)$$

Where e_t is the standardized log return, and the constants a, b and c are given by $a = 4\beta c\left(\frac{\nu-2}{\nu-1}\right)$, $b^2 = 1+3\beta^2-a^2$, and $c = \frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{\pi(\nu-2)}\Gamma(\frac{\nu}{2})}$. The density function has a mode of $-a/b$, a mean of zero, and a unit variance. The density function is skewed to the right when $\beta>0$, and vice-versa when $\beta>0$. The Skewed *t* distribution specializes to the standard Student's *t* distribution by setting the parameter $\beta>0$.

(iii) Normal Inverse Gaussian Distribution (NIG)

$$f(e_t \mid \mu, \alpha, \beta, \delta) = \frac{\alpha\delta K_1(\alpha\sqrt{\delta^2+(e_t-\mu)^2})}{\pi\sqrt{\delta^2+(e_t-\mu)^2}}\exp(\delta\sqrt{\alpha^2-\beta^2}+\beta(e_t-\mu)), \quad (3)$$

Where K_λ (.) is the modified Bessel function of the third kind and index $\lambda=0$ and $\alpha>0$. The NIG distribution is specified as in Prause (1997). The NIG distribution is normalized by setting $\mu = -\delta\beta/\sqrt{\alpha^2-\beta^2}$ and $\delta = \frac{\left(\sqrt{\alpha^2-\beta^2}\right)^3}{\alpha^2}$, which implies $E(e_t)=0$ and $Var(e_t)=1$.

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(iv) Generalized Hyperbolic Distribution

$$f(e_t \mid p,b,g) = \frac{g^p}{\sqrt{2\pi}\left(b^2+g^2\right)^{\frac{1}{2}\left(p-\frac{1}{2}\right)} d(p,b,g) K_p(g)} q\left(\frac{e_t - m(p,b,g)}{d(p,b,g)};\ p,b,g\right), \quad (4)$$

Where $\tilde{R}_n = \frac{K_{n+p}(g)}{g^n K_p(g)}$, $d(p,b,g) = \left[\tilde{R}_1 + b^2\left\{\tilde{R}_2 - \tilde{R}_1^2\right\}\right]^{-\frac{1}{2}} \geq 0$, and $m(p,b,g) = -b\,d(p,b,g)\tilde{R}_1$.

p, b and g are parameters. The generalized hyperbolic distribution is a standardized version of Prause (1997).

III. Data

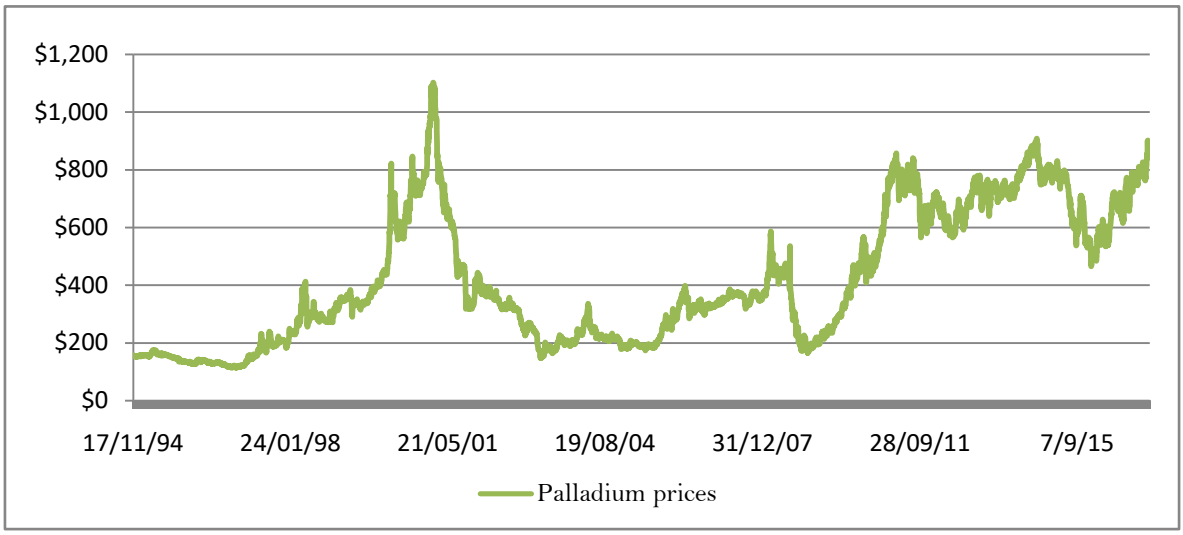

Figure 1: Daily Palladium Prices

We collected the data from Bloomberg, which sourced the data from the London Platinum and Palladium Market (LPPM). The LPPM is the most important over-the-counter trading market for platinum and palladium and one of the world's major commodity trading associations. The trade in LPPM was established in the early 20th century, typically by existing dealers of gold and silver. Our data covers the period from November 17, 1994 to June 30, 2017 with total 6459 observations. Figure 1 illustrates the daily palladium spot prices in the LPPM. The figure indicates the palladium spot prices have never researched the peak level of $1102.5 per ounce in January 27, 2001 in the last decade. Figure 2 illustrates the dynamics of the palladium spot returns. Except the two negative and positive spikes in the recent financial crisis, the return series exhibits several similar stylized facts as in other types of asset return series as in Cont (2001): no return prediction, fait tails, volatility clustering, conditional fat tails, and so on.

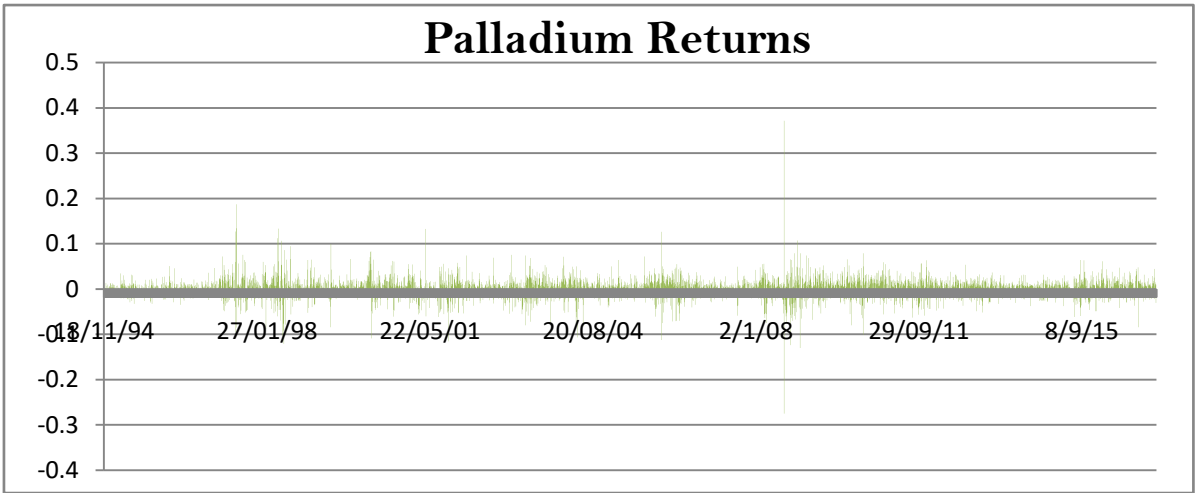

Figure 2: Daily Palladium Returns

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Table 1 exhibits basic statistics of the daily palladium spot returns. The results show the daily palladium spot returns are leptokurtotic and positively skewed. The extreme downside move is slightly less than the extreme upside move, which is at odds with most of other asset returns which are more likely to exhibit negative skewness.

Table 1: Descriptive Statistics

Min	Max	Mean	Std	Skewness	Kurtosis
-27.46%	37.16%	0.05%	2.01%	0.66	28.84

Figure 3 is the histogram of the raw data. We fit the returns by the normal distribution and the figure clearly exhibits significant heavy tails.

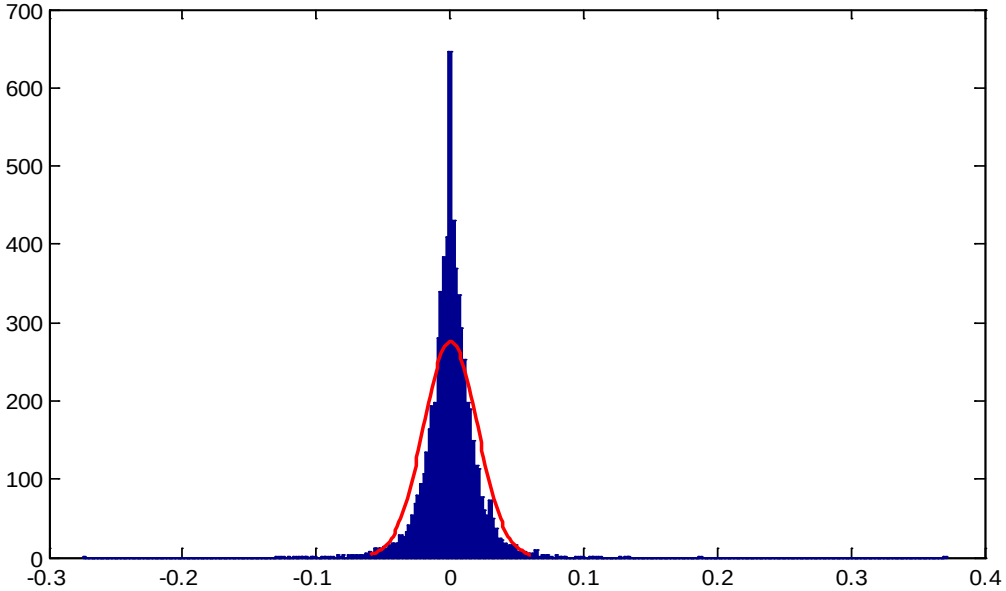

Figure 3: Histogram of daily palladium spot returns

IV. Empirical Results

a) Parameters Estimation

The raw return series is normalized to allow zero mean and unit standard deviation. We use the maximum likelihood estimation (MLE) method to fit the series and the estimation results of the key parameters are given in Table 2. All the parameters are significantly different from zero at 10% significance level.

Table 2: Estimated Values of Key Parameters

	Normal	Student's t	Skewed t	NIG	Generalized Hyperbolic
Symmetric	Y	Y	N	N	N
Fat-tailed	N	Y	Y	Y	Y
Estimated Parameters		Nu=2.76	Nu=2.83; beta=0.023	alpha=1.31; beta=0.017	p=-123; b=-.032; g=0.07

b) Goodness of Fit

There are many different types of criteria for statistical distributions selection. In this paper, we focus on the following different criteria for the selection of the four heavy-tailed distributions and the benchmark normal distribution in fitting the daily returns: (i) Kolmogorov-Smirnov statistic; (ii) Cramer-von Mises criterion; (iii) Anderson-Darling test; and (iv) Akaike information criterion (AIC). For detailed discussions about the pros and cons of the four different criteria, one could refer to Huber-Carol, et al. (2002) and Taeger and Kuhnt (2014).

(i) Kolmogorov-Smirnov statistic is defined as the maximum deviation between empirical CDF (cumulative distribution function) $F_n(x)$ and tested CDF $F(x)$:

$$D_n = \sup_x |F_n(x) - F(x)|, \qquad (5)$$

Where,

$$F_n(x) = \frac{1}{n}\sum_{i=1}^{n} I_{[-\infty,x]}(X_i).$$

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(ii) Cramer-von Mises criterion is defined as the average squared deviation between empirical CDF and tested CDF:

$$T = n\int_{-\infty}^{\infty}[F_n(x) - F(x)]^2 dF(x) = \frac{1}{12n} + \sum_{i=1}^{n}\left[\frac{2i-1}{2n} - F_n(x_i)\right]^2, \quad (6)$$

(iii) Anderson-Darling test is defined as the weighted-average squared deviation between empirical CDF and tested CDF:

$$A = n\int_{-\infty}^{\infty}\frac{(F_n(x) - F(x))^2}{F(x)(1-F(x))}dF(x),$$

And the formula for the test statistic A to assess if data comes from a tested distribution is given by:

$$A^2 = -n - \sum_{i=1}^{n}\frac{2i-1}{n}\left[\ln(F(x_i)) + \ln(1-F(x_i))\right]. \quad (7)$$

(iv) Akaike information criterion (AIC) is defined as:

$$AIC = -2k - 2\ln(L), \quad (8)$$

Where L is the maximum value of the likelihood function for the model, and k is the number of estimated parameters in the model.

The comparison results are showed in Table 3, indicating the Skewed t distribution has the best goodness of fit compared with other selected types of distribution, followed by the generalized hyperbolic distribution, and the Student's t distribution.

Table 3: Comparison of selected types of distribution

	Normal	Student's t	Skewed t	NIG	Generalized Hyperbolic
K-S Test	0.097	0.092	0.090	0.092	0.091
Cv-M Test	0.186	0.182	0.180	0.182	0.181
A-D Test	2.503	2.352	2.297	2.338	2.311
AIC	25621	24538	24139	24750	24374

c) Hypothetical Rare Scenarios

Since the main objective of this paper is to develop effective quantitative risk manage tools, in this section we take advantage of the widely-used tool, Value at Risk (VaR). The concept of VaR was originally developed by JP Morgan in the early 1990s, and soon emerged as a standard quantitative risk management tool in the industry. VaR is defined as: for a given position, time horizon, and probability *p*, the *p* VaR is defined as a threshold loss value, such that the probability that the loss on the position over the given time horizon exceeds this value is *p*.with the estimated parameters in Section 4.1, we calculate VaRs for different confidence levels:

$$VaR_\alpha(e_t) = \inf\{e \in \mathbb{R} : P(e_t > e) \le 1-\alpha\}, \quad (9)$$

Where $\alpha \in (0,1)$ is the confidence level. We select the following levels for downside moves: {99.99%, 99.95%, 99.9%, 99.5%, 99.0%}, and for upside moves: {0.01%, 0.05%, 0.1%, 0.5%, 1.0%}. From Equation (9), the hypothetical rare scenarios based on the VaR levels are given as in Table 4. Table 4 indicates that the Skewed t distribution has the closest VaRs to the nonparametric historical VaRs compared with other types of distributions.

Table 4: Scenarios for daily palladium spot return shocks

Left Tail					
Confidence	99.99%	99.95%	99.90%	99.50%	99.00%
Empirical	-27.97%	-26.33%	-22.79%	-19.68%	-17.82%
Normal	-20.06%	-18.84%	-18.21%	-17.19%	-16.56%
T	-27.17%	-25.28%	-23.88%	-21.18%	-19.29%
Skewed T	-27.76%	-26.43%	-22.27%	-19.89%	-18.14%
NIG	-25.14%	-23.95%	-22.02%	-20.94%	-19.19%
GH	-27.24%	-25.56%	-23.21%	-20.45%	-18.49%
Right Tail					
Confidence	0.01%	0.05%	0.10%	0.50%	1.00%
Empirical	34.73%	30.50%	29.00%	25.80%	23.92%
Normal	20.06%	18.84%	18.21%	17.19%	16.56%
T	27.17%	25.28%	23.88%	21.18%	19.29%
Skewed T	35.53%	31.16%	29.56%	26.09%	24.21%
NIG	36.75%	34.26%	31.11%	28.91%	26.84%
GH	38.40%	34.69%	31.68%	28.72%	25.66%

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V. Conclusions

The past decade has witnessed the rapid increase of the world palladium market. Thus, it is even more important to develop effective quantitative risk management tools at this moment. In this paper, we investigate five different types of widely-used statistical distributions and employ the industry standard risk measurement, Value at Risk, for risk management of daily palladium spot returns. We first apply four different criteria to compare the goodness of fit of the five distributions, and then calculate the VaRs based on the parameters estimated from the first step. Our results indicate the Skewed *t* distribution has the best in-sample fitting and generate VaR values closest to the nonparametric historical VaR values. There is one potential direction for further research. In Figure 2, we observed the volatility clustering phenomenon, which is usually captured by the generalized autoregressive conditional heteroskedasticity (GARCH) process in financial econometric modeling as in Guo (2017b). It would be interesting to incorporate the GARCH model into our current setting to discuss the relevant results.

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